

VIC SKEPTICS

Logic and Maths Puzzles 109 August 2021

1.

HOUSE-SOLD HINTS

Four Melbourne inner-suburban houses were sold at auction last week. The vendors were the Collins family, the Armstrongs, the Frasers and the Rays. The houses are located in Collingwood, Albert Park, Fitzroy and Richmond, and each had a dissimilar appearance to the other three.

From the clues and grid provided, determine the previous owners of each house, the suburb in which it was situated, its appearance and the price it sold for.

	Collingwood	Albert Park	Fitzroy	Richmond	Timber	Cladding	Brick	Adobe	\$ 900,000	\$ 1.0 m	\$ 1.1 m	\$ 1.2 m
Collins												
Armstrong												
Fraser												
Ray												
\$ 900,000												
\$ 1.0 m												
\$ 1.1 m												
\$ 1.2 m												
Timber												
Cladding												
Brick												
Adobe												

(i). Only one of the families has the same initial as the name of the suburb in which their house was situated.

(ii). The Collins's house didn't fetch \$1.1 million dollars. It is either the adobe style house or it is in Fitzroy.

(iii). The Armstrong's house wasn't in Richmond, and wasn't the house which kept its original brick appearance.

(iv). The Albert Park home sold for either 1.1 or 1.2 million dollars.

(v). The Rays sold either the Fitzroy house or the one covered in vinyl cladding.

(vi) Of the Fraser's house and the one in Richmond, one sold for \$1.2 million, while the other was adobe style.

(vii) The Collingwood home wasn't covered in vinyl cladding.

(viii) The Fitzroy property's going price was less than that of the one in Richmond.

(ix) The brick house sold for one million dollars; or it was the one the Rays put on the market.

(x) The house that maintained its original timber appearance was more expensive than the one covered in cladding.

2.

WHAT'S THE RULE?

The numbers on the right-hand column of this table are each formed from the corresponding number in the left-hand column using the same rule. **x = ?**

9	→	5
36	→	8
169	→	15
256	→	x

Figure It Out



3.

Three days ago, yesterday was the day before Sunday. What day will it be tomorrow?

4.

Fred's Bed

Fred bought a camp bed. The price was \$65.

Fred, being colour-blind gave the salesman what he thought were two \$50 notes, (actually two \$100 notes).

The salesman was sleepy and also colour-blind, so he gave Fred what he thought were seven \$5 notes (actually seven \$20 notes) in change.

Who ended up better off than they should, and by how much?

5.

Parking Possibilities



There's a car park next to the Maths Faculty building at the University. It only costs 30 cents to park all day.

You can make up the 30 cents with any combination of 5 cent, 10 cent and 20 coins, (including just 6 X 5c coins or just 3 X 10c coins)

However, the order in which you insert the coins is important. For example, 5c, then 10c, 5c, 5c and 5c

is different to

10c, then 5c, 5c, 5c and 5c.

The meters will only allow each person to park once for each possible combination of 5c, 10c and 20c coins making up 30 cents.

Once each person exhausts all possible combinations, they'll have to park in a more expensive area.

What is the greatest number of days for which any one person may park for just 30 cents a day?

6.

MASTERMIND

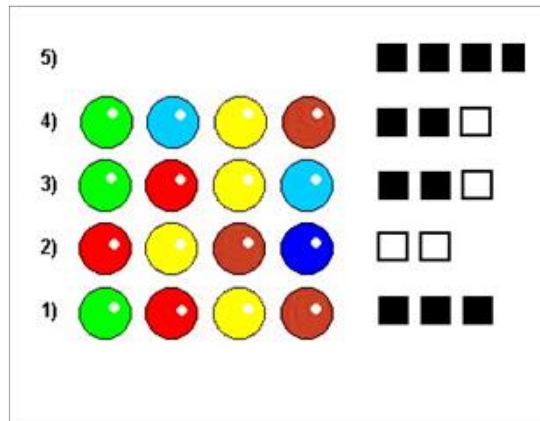
In the game of Mastermind, a board is sectioned off into rows, each row having four slots in which pegs can be inserted. There are 6 different colours of pegs: green, red, yellow, brown, dark-blue, light-blue. Player (A) makes up some arrangement of four pegs along a row; Player (B) tries to guess what this arrangement is. For every guess that B makes, A responds by putting black and/or white keypegs right next to A's guess; as follows:

Black keypeg = one of B's pegs is the correct colour and in the correct position

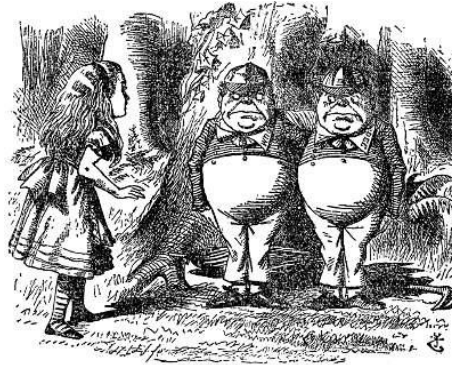
White keypeg = one of B's pegs is the correct colour but in the wrong position

So if B manages to guess all four colours and positions correctly, A will respond with four black keypegs, and the game is over.

Here's a completed game of Mastermind. B was able to determine A's arrangement using only five guesses. What's A's arrangement? (State the four colours, left to right).



7. Go ask, Alice! (Again)



We met Tweedledum and Tweedledee last month (July 2021). They reappear in this slightly different problem.

Tweedledum and Tweedledee ("Dum" and "Dee" for short) look alike, but Dum lies on Monday, Tuesday, and Wednesday, whereas Dee lies on Thursday, Friday, and Saturday. They both tell the truth on Sunday. You come upon the two of them on four different days of the week, and they make the following statements. In each case, determine who is X and who is Y on that day, and what day it is. The first one is an example.

	statements	Which is which?	What day is it?
example	X says: <i>I will lie tomorrow.</i> Y says: <i>I lied yesterday, and I will lie tomorrow.</i>	X is "Dee" Y is "Dum"	Wednesday
Day (a)	X says: <i>Today is not Sunday.</i> Y says: <i>Today is not Monday</i>		
Day (b)	X says: <i>Today is Sunday</i> Y says: <i>Today is Tuesday</i>		
Day (c)	X says: <i>I am Tweedledum</i> Y says: <i>I am Tweedledee</i>		



Figure It Out

8.



Census Taker: *How many children do you have?*

Woman: *Three.*

Census Taker: *And their ages?*

Woman: *One is seven, one is five, one is two.*

Census Taker: *Is at least one of them a girl?*

Woman: *Yes.*

Given that information, what is the probability that her *other* two children are also girls?

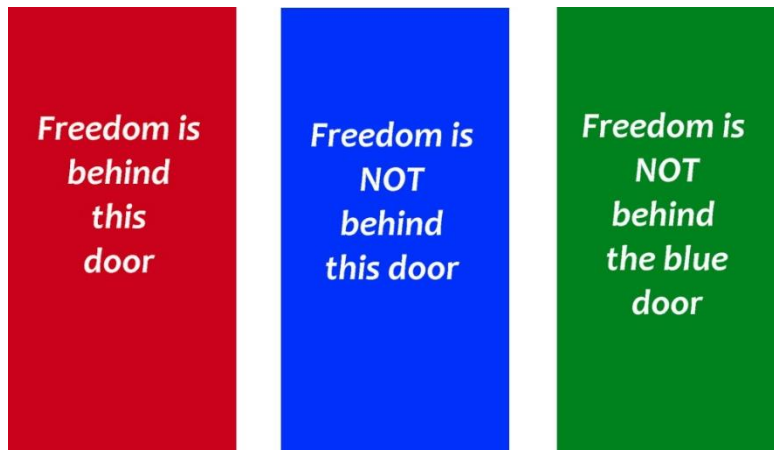
(For this problem assume each baby has a 50/50 chance to be born either a boy or a girl.)

9. Dungeon and Dragons

You are trapped in a dungeon which has three exit doors as shown.

Behind one of the doors lies freedom. Behind each of the other two, however, lurks a death-dealing dragon.

Given the fact that at LEAST ONE of the three statements on the three doors is true and at LEAST ONE of them is false, which colour door leads to freedom?



10.

Up Where, Cazaly?



April, Blanche, and Carrie are enthusiastic AFL supporters. One barracks for the Sydney Swans, another is an Adelaide Crows supporter and the third follows the Richmond Tigers. If only one of the following four statements is true, link each of the women to the team she supports.

- Carrie follows the Crows.
- Carrie does not support the Tigers.
- Blanche does not support the Tigers.
- Blanche does not barrack for the Swans.

Answers on the next page.

Answers: *(Worked solutions begin on the next page.)*

1. The Collins family sold their adobe-style house in Richmond for \$1.0 million.
The Armstrong family sold their vinyl-cladded house in Albert Park for \$1.1 million.
The Fraser family sold their timber house in Collingwood for \$1.2 million.
The Ray family sold their brick house in Fitzroy for \$900,000.
2. $x = 18$
3. Thursday
4. Fred was better off by \$5
5. 18 days
6. In order, left to right: Green, red, light blue, brown
7. Day (a) X is Dee, Y is Dum It's Monday
Day (b) X is Dum, Y is Dee It's Tuesday
Day (c) X is Dum, Y is Dee It's Sunday
8. 1 in 7
9. The green door
10. April – Adelaide Crows; Blanche – Sydney Swans; Carrie – Richmond Tigers

From clue (ii), the Collins house is either adobe or brick. It follows that the Richmond house (Collins or Ray) and the timber house (Armstrong or Fraser) are different houses.

	Collingwood	Albert Park	Fitzroy	Richmond	Timber	Cladding	Brick	Adobe	\$ 900,000	\$ 1.0 m	\$ 1.1 m	\$ 1.2 m
Collins					X	X					X	
Armstrong			X	X			X					
Fraser				X								
Ray					X				X			X
\$ 900,000	X		X	X	X							
\$ 1.0 m	X				X							
\$ 1.1 m					X							
\$ 1.2 m			X		✓	X	X	X				
Timber			X	X								
Cladding	X		X									
Brick	X	X	✓	X								
Adobe			X									

From clue (v), the Rays did not sell the Collingwood house, and the house they did sell was either brick or cladding.

	Collingwood	Albert Park	Fitzroy	Richmond	Timber	Cladding	Brick	Adobe	\$ 900,000	\$ 1.0 m	\$ 1.1 m	\$ 1.2 m
Collins					X	X					X	X
Armstrong			X	X			X					
Fraser				X								
Ray	X				X			X	X			X
\$ 900,000	X		X	X	X							
\$ 1.0 m	X				X							
\$ 1.1 m					X							
\$ 1.2 m			X	X	✓	X	X	X				
Timber			X	X								
Cladding	X		X									
Brick	X	X	✓	X								
Adobe			X									

Consider clues (iii) and (vi) together. We have established that the vendors of the \$1.2 m house were either the Armstrongs or the Frasers. From clue (vi), IF the Fraser's house sold for \$1.2 m, then it is timber and situated in Collingwood or Albert Park; the Richmond house is adobe style, and sold for \$1.0 m or \$1.1 m. IF the Richmond house sold for \$1.2 m, then it is Armstrongs. However, clue (iii) says that the Armstrongs did NOT have the Richmond house. Therefore, the Fraser house sold for \$1.2 m and the Richmond house is adobe style. After recording this information, the table can be completed to a conclusion (next page)

	Collingwood	Albert Park	Fitzroy	Richmond	Timber	Cladding	Brick	Adobe	\$ 900,000	\$ 1.0 m	\$ 1.1 m	\$ 1.2 m
Collins	X	X	X	✓	X	X	X	✓	X	✓	X	X
Armstrong	X	✓	X	X	X	✓	X	X	X	X	✓	X
Fraser	✓	X	X	X	✓	X	X	X	X	X	X	✓
Ray	X	X	✓	X	X	X	✓	X	✓	X	X	X
\$ 900,000	X	X	✓	X	X	X	✓	X				
\$ 1.0 m	X	X	X	✓	X	X	X	✓				
\$ 1.1 m	X	✓	X	X	X	✓	X	X				
\$ 1.2 m	✓	X	X	X	✓	X	X	X				
Timber	✓	X	X	X								
Cladding	X	✓	X	X								
Brick	X	X	✓	X								
Adobe	X	X	X	✓								

Checking against clue (i): The only family whose initials coincide with the location of their former house is Armstrong (Albert Park).

- By inspection, the relationship between the left-hand values and the right-hand values is not linear. As the numbers in the left-hand column are all perfect squares, try taking their square roots for comparison with the numbers in the right-hand column.

l.h.	$\sqrt{\text{l.h.}}$	r.h.
9	3	5
36	6	8
169	13	15
256	16	x

The relationship is clear. Each right-hand number can be generated by taking the square root of its corresponding left-hand number and adding 2.

$$\begin{aligned}
 X &= \sqrt{256} + 2 \\
 &= 16 + 2 \\
 &= 18
 \end{aligned}$$

- Three days ago, yesterday was the day before Sunday. What day will it be tomorrow?
Three days ago yesterday was four days ago. The day before Sunday is Saturday. Saturday was four days ago. Today is Wednesday. Tomorrow is Thursday.
- Fred gave the salesman $2 \times \$100 = \200 .
The salesman returned $7 \times \$20 = \140 in change.
Fred therefore only paid \$60 for his bed and is \$5 better off than he should be.
- There are 18 possible ways of creating 30c using 5c, 10c and 20c coins in all possible combinations and orders of insertion into the meter. (See table next page)

	order						
	1 st	2 nd	3 rd	4 th	5 th	6 th	
Possible combinations using all 5 c coins	5	5	5	5	5	5	1
Possible combinations using one 10c coin and four 5 c coins	10	5	5	5	5		5
	5	10	5	5	5		
	5	5	10	5	5		
	5	5	5	10	5		
	5	5	5	5	10		
Possible combinations using two 10c coins and two 5c coins	5	5	10	10			6
	5	10	5	10			
	5	10	10	5			
	10	5	5	10			
	10	5	10	5			
	10	10	5	5			
Possible combinations using all 10 c coins	10	10	10				1
Possible combinations using a 20c coin	20	10					5
	10	20					
	20	5	5				
	5	20	5				
	5	5	20				
Total combinations							18

6. Receiving three black pegs for his first guess tells B that three of the pegs are the right colour and in the right position. The fourth peg is the wrong colour.

For the second guess, B has swapped a dark blue peg (in) for a green peg (out) but has only received two white pegs in reply. This tells him:

- (a) There is at least one green peg in the correct answer, and it's in the left-most position.
- (b) There are no dark blue pegs in the correct answer
- (c) Two of the colours red, yellow and brown are correct, in the positions of the first guess

For the third guess, B has gone back to the colours and positions of his first guess, but has replaced the brown peg with a light blue peg. He receives two black pegs and one white peg in reply. This tells him:

- (a) Both a brown peg and a light blue peg are in the correct mix.
- (b) The brown peg is in the right-most position.
- (c) The light blue peg is either second from left or second from right. If the light blue peg is second from left, then there's no red peg and the yellow peg is correct in the second from right position. However, if the light blue peg is second from right, then there's no yellow peg and the red peg is correct in the second from left position.

For the fourth guess, B is testing to see whether the alternative of the light blue peg in the second from left position is the correct one. He knows that there's a green peg in the left-most position and a brown peg in the right-most position. The fact that he receives only two black and one white peg in response tells him he chose the wrong alternative.

His fifth guess will be, (left-to-right) green, red, light blue and brown, for which he will receive four black pegs.

7.

Day (a)	X says: <i>Today is not Sunday.</i> Y says: <i>Today is not Monday</i>	It's not Sunday, because they both tell the truth on Sunday. X is therefore telling the truth, so Y must be lying. It's Monday. X is Dee, Y is Dum.
Day (b)	X says: <i>Today is Sunday</i> Y says: <i>Today is Tuesday</i>	It can't be Sunday, because they'd both be telling the truth and agreeing it's Sunday. Therefore, X is lying, and Y is telling the truth. It's Tuesday. X is Dum, Y is Dee
Day (c)	X says: <i>I am Tweedledum</i> Y says: <i>I am Tweedledee</i>	Either they are both lying (not possible), so they are both telling the truth. It's Sunday, X is Dum and Y is Dee.

8. For any woman to have three children of different ages, there are eight equally likely distributions with respect to age and gender, viz.

All three are boys.

The eldest and middle child are boys, youngest child is a girl.

The eldest child is a boy, middle child is a girl, youngest child is a boy.

The eldest child is a girl, middle and youngest child are boys

The eldest and middle child are girls, youngest child is a boy.

The eldest child is a girl, middle child is a boy, youngest child is a girl.

The eldest child is a boy, middle and youngest child are girls.

All three are girls.

However, if you know that at least one of the children is a girl, that eliminates "three boys" and leaves only seven equally likely outcomes, one of which is "three girls".

9. Assume Freedom is behind the red door. All three doors would then have true statements which we know is not possible since one of them must be false.

Assume Freedom is behind the blue door. All three doors would then have false statements which we know is not possible since one of them must be true.

Freedom is therefore behind the green door. The blue door and the green door have true statements and the red door has a false statement.

10. "Carrie does not support the Tigers."

"Blanche does not support the Tigers."

At least one of those statements must be false. They can't both be false, because then both Carrie and Blanche would be Tiger fans.

So either Carrie is the Tiger supporter or Blanche is the Tiger supporter. Since one of those statements is true, the other three must be false, including:

"Carrie follows the Crows". Carrie doesn't follow the Crows.

"Blanche does not barrack for the Swans". Blanche DOES barrack for the Swans.

Therefore "Blanche does not support the Tigers" is the lone true statement.

"Carrie does not support the Tigers." Is false. Carrie DOES support the Tigers.

By elimination, April is the Crows fan.