VIC SKEPTICS

Logic and Maths Puzzles 108 July 2021

1. ARE WE THERE YET?

Four families went on holiday today, traveling in their SUVs. From the clues provided, and with the help of the grid, work out the number of people in each family, the vehicle they travelled in and the time they left home.

	8:00	9:00	10:00	11:00	Kluger	Everest	Odyssey	CX9	4	5	6	7
Brown												
Green												
Smith												
White												
4												
5												
6												
7												
Kluger												
Everest												
Odyssey												
CX9												

(i). The Ford Everest carried 6 people.

(ii). The White family set off one hour after the family in the Toyota Kluger, and one hour earlier than the largest family.

(iii). The Green family owns either the Ford Everest or the Mazda CX9. They have one more family member than the Browns.

(iv). The Smiths don't own the Honda Odyssey and aren't the smallest family.

(v). The smallest family doesn't own the Mazda CX9.

(vi). If the Greens left at 9:00, then the largest family left at 10:00.

(vii). If the family with the Ford Everest left at 8:00, then there are 7 people in the Smith family

2. Figure It Out

Every member of a certain cricket team is given his own unique number, a practice which started when the team played its first official match, with the then captain being given the number 1, the vice-captain number 2 and so on. Those players each received one ceremonial cap bearing their team number sewn on with cloth digits.

Since then, each new player who makes the team has been presented with one ceremonial cap with his team number sewn on in separate cloth digits. (The first person to require three digits on his cap was therefore the hundredth player to make the team.)

So far, 585 digits have been sewn on to the ceremonial caps presented to team members. What is the team number of the latest player to make the team?

3. Figure It Out

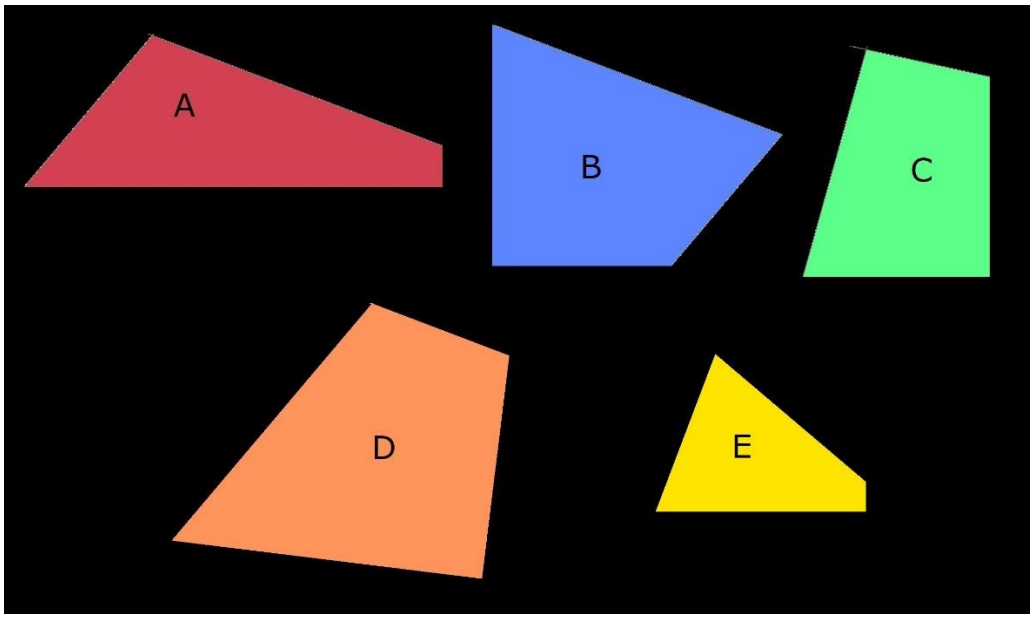
There were 120 chocolates in a box. The box was passed along a row of people.

The first person took one chocolate. Each person down the row took more chocolates than the person before, until the box was empty.

What is the largest number of people that could have been in the row?

4. # ODD ONE OUT

Four of the five shapes below fit together to form a square. Which is the odd one out?



5. # Figure It Out 

A solo dice game is played where, on each turn, a normal pair of dice is rolled. The score is calculated by taking the **product**, rather than the sum, of the two numbers shown on the dice.

On a particular game, the score for the second roll is 5 more than the score for the first; the score for the third roll is 6 less than that of the second; the score for the fourth roll is 11 more than that of the third; and the score for the fifth roll is 8 less than that of the fourth. What was the score for each of these five throws? Write your answer as five numbers in the correct order.

6. # WHAT's THE RULE?

In each of the four cases below, the number on the right-hand is generated from the corresponding number in the left-hand using the same rule.
Find the rule, and thus determine the value of x.

14 → 3

46 → 5

238 → 11

388 → x

7. **Go ask, Alice!**

Tweedledum lies every Monday, Tuesday and Wednesday and on the other days he speaks the truth.

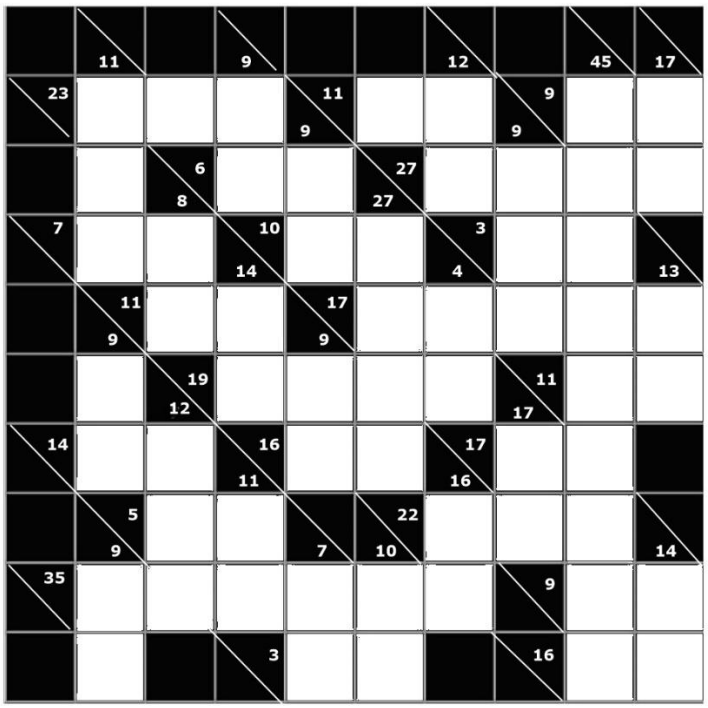
Tweedledee lies on Thursdays, Fridays and Saturdays, and on the other days of the week he speaks the truth.

"Yesterday I was lying," says Tweedledum.

"So was I," says Tweedledee.

What day is it?

8. **CROSS SUMS**



This is a Cross Sum puzzle.
It is solved by inserting one digit from 1 to 9 into each white square so that each horizontal or vertical string of digits adds up to a given target number.
For example 

means "find two digits that add up to 7"
and this

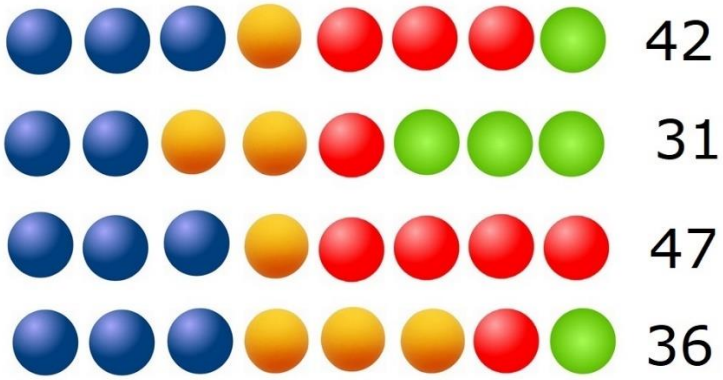
means "find three digits that add up to 9"

The digits you use must agree horizontally and vertically, just like the letters in a crossword puzzle

You can only use each digit once in any string. For example, the target number 16 can be made up from 9 and 7, NOT 8 and 8.

9. **VALUABALLS**

The diagram at the top of the next page shows four rows of coloured balls: (blue, yellow, red and green).
Each ball has a whole number value from 1 to 10 inclusive. All balls of the same colour have the same value. Each colour has a different value to that of the other three colours. The number at the end of each row is the total value for that row.



Determine the numerical value for each colour.

10. **GAME of THRONES**

A duke is arrested on a charge of high treason against his king, and is brought to trial. The Lord Chamberlain tells him to make a statement about his guilt or innocence.

If the statement is true, he will be treated as an aristocrat and beheaded.

However, if the statement is deemed to be false, he will be hanged like an ordinary commoner.

"What if you can't tell whether my statement is true or false?" asks the duke.

"I suppose we'll have to let you go", answers the Lord Chamberlain.

Shortly after, the duke rides off to freedom. What was his statement?

Answers on the next page.

Answers: *(Worked solutions begin on the next page.)*

1. The White family of four people left at 10:00 am in a Honda Odyssey.
   The Brown family of five people left at 9:00 am in a Toyota Kluger.
   The Green family of six people left at 8:00 am in a Ford Everest.
   The Smith family of seven people left at 11:00 am in a Mazda CX9.

2. 231

3. 15

4. Shape C

5. (In order) 10, 15, 9, 20, 12

6. The rule: Add 4 to the left-hand number
   Divide by 2
   Take the square root

   OR

$$x = \sqrt{\frac{(y+4)}{2}}$$

$\boldsymbol{x}$ = 14

7. Thursday.

8.

	11		9			12		45	17
23	6	9	8	11 / 9	3	8	9 / 9	1	8
	3	6 / 8	1	5	27 / 27	4	6	8	9
7	8	5	10 / 14	4	6	3 / 4	1	2	13
	11 / 9	3	8	17 / 9	4	1	2	3	7
	1	19 / 12	6	2	8	3	11 / 17	5	6
14	8	6	16 / 11	7	9	17 / 16	8	9	
	5 / 9	1	4	7	22 / 10	7	9	6	14
35	8	5	7	6	8	9	9	4	5
	1		3	1	2		16	7	9

9. blue =5, yellow = 4, red = 7, green =2

10. "You'll have to hang me."

SOLUTIONS:

1. Begin by copying all the direct information supplied by clues (i) to (v) into the grid.

	8:00	9:00	10:00	11:00	Kluger	Everest	Odyssey	CX9	4	5	6	7
Brown												X
Green					X		X		X			
Smith							X		X			
White	X			X	X							X
4						X		X				
5						X						
6					X	✓	X	X				
7	X	X			X	X						
Kluger			X	X								
Everest												
Odyssey												
CX9												

The Odyssey is owned by either the Brown or White families. Neither the Brown nor the White families are the largest (7 people); therefore the Odyssey is not the car carrying 7 people. That only leaves the CX9 as the car carrying 7 people.

	8:00	9:00	10:00	11:00	Kluger	Everest	Odyssey	CX9	4	5	6	7
Brown								X				X
Green					X		X		X			
Smith							X		X			
White	X			X	X			X				X
4						X		X				
5						X		X				
6					X	✓	X	X				
7	X	X			X	X	X	✓				
Kluger			X	X								
Everest												
Odyssey												
CX9	X	X										

The Green family are in the Everest (6 people) or the CX9 (7 people). If the Greens are 6 people, then the Browns are 5. If the Greens are 7 people, then the Browns are 6. (Clue iii). Since either the Greens or the Browns are the 6-person family, neither the Smiths nor the Whites can be the 6-person family. Since the Browns are not the 4-person family, that only leaves the Whites as the 4-person family.

The Whites don't own the Kluger. As they are the 4-person family, it must mean that they own the Odyssey.

	8:00	9:00	10:00	11:00	Kluger	Everest	Odyssey	CX9	4	5	6	7
Brown							X	X	X			X
Green					X		X		X	X		
Smith							X		X		X	
White	X			X	X	X	✓	X	✓	X	X	X
4	X			X	X	X	✓	X				
5			X	X	✓	X	X	X				
6					X	✓	X	X				
7	X	X			X	X	X	✓				
Kluger			X	X								
Everest												
Odyssey	X			X								
CX9	X	X										

Now consider clue (vi). *If the Greens left at 9:00, then the largest family left at 10:00.*

If the Greens left at 9:00, then the Whites left at 10:00 (the only possibility remaining). However, the Whites are NOT the largest family. Therefore, the Greens did NOT leave at 9:00 and the largest family did NOT leave at 10:00. By default, the largest family left at 11:00.

	8:00	9:00	10:00	11:00	Kluger	Everest	Odyssey	CX9	4	5	6	7
Brown				X			X	X	X			X
Green		X			X		X		X	X		
Smith							X		X		X	
White	X			X	X	X	✓	X	✓	X	X	X
4	X			X		X	✓	X				
5				X		X	X	X				
6				X	X	✓	X	X				
7	X	X	X	✓	X	X	X	✓				
Kluger			X	X								
Everest				X								
Odyssey	X			X								
CX9	X	X	X	✓								

Revisiting clue (ii). The White family set off one hour after the family in the Toyota Kluger, and one hour earlier than the largest family.

	8:00	9:00	10:00	11:00	Kluger	Everest	Odyssey	CX9	4	5	6	7
Brown			X	X			X	X	X			X
Green		X	X		X		X		X	X		
Smith			X				X		X		X	
White	X	X	✓	X	X	X	✓	X	✓	X	X	X
4	X	X	✓	X	X	X	✓	X				
5	X	✓	X	X	✓	X	X	X				
6	✓	X	X	X	X	✓	X	X				
7	X	X	X	✓	X	X	X	✓				
Kluger	X	✓	X	X								
Everest	✓	X	X	X								
Odyssey	X	X	✓	X								
CX9	X	X	X	✓								

Now consider clue (vii). *If the family with the Ford Everest left at 8:00, then there are 7 people in the Smith family.* We've established that the family with the Ford Everest did indeed leave at 8:00, therefore the largest family is the Smith family.

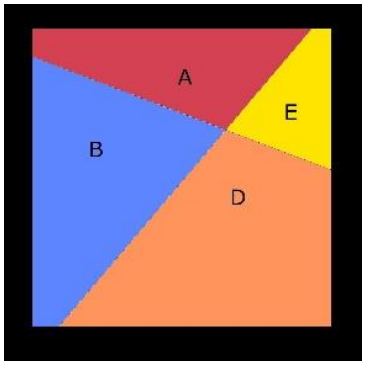
	8:00	9:00	10:00	11:00	Kluger	Everest	Odyssey	CX9	4	5	6	7
Brown	X	✓	X	X	✓	X	X	X	X	✓	X	X
Green	✓	X	X	X	X	✓	X	X	X	X	✓	X
Smith	X	X	X	✓	X	X	X	✓	X	X	X	✓
White	X	X	✓	X	X	X	✓	X	✓	X	X	X
4	X	X	✓	X	X	X	✓	X				
5	X	✓	X	X	✓	X	X	X				
6	✓	X	X	X	X	✓	X	X				
7	X	X	X	✓	X	X	X	✓				
Kluger	X	✓	X	X								
Everest	✓	X	X	X								
Odyssey	X	X	✓	X								
CX9	X	X	X	✓								

2. Players 1 to 9 required 1 digit each. That's 9 digits.
   The ninety players with numbers 10 to 99 required 2 digits each. That's 180 digits
   Players 100 to 999 require 3 digits each.
   The players with numbers 100 and above account for (585 – 189) or 396 of the digits used. That must be 396/3 = 132 players. The latest player to receive his cap is player number (99 + 132) or player number 231

3. The greatest number of people in the row will occur if each person takes just one more chocolate than the previous person.

   1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 = 120

4. This is really a test of spatial awareness



5. What are the possible scores from the product of two dice?

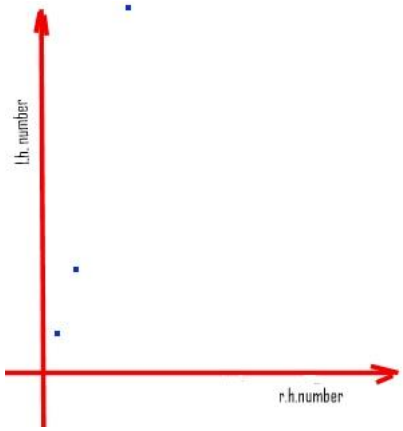
Dice 2 \ Dice 1	1	2	3	4	5	6
1	**1**	**2**	**3**	**4**	**5**	**6**
2	**2**	**4**	**6**	**8**	**10**	**12**
3	**3**	**6**	**9**	**12**	**15**	**18**
4	**4**	**8**	**12**	**16**	**20**	**24**
5	**5**	**10**	**15**	**20**	**25**	**30**
6	**6**	**12**	**18**	**24**	**30**	**36**

One approach is to list all possible scores for the first roll, then eliminate roll by roll as the clue for each roll does not allow for certain scores.

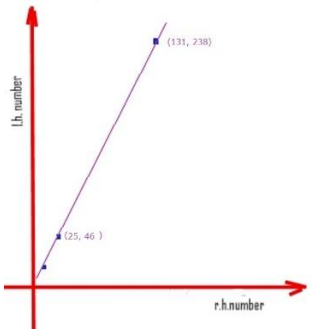
1st roll	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~8~~	~~9~~	10	~~12~~	~~15~~	~~16~~	~~18~~	20	~~24~~	~~25~~	~~30~~	36
2nd roll (1st roll + 5)	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~	~~11~~	~~13~~	~~14~~	15	~~17~~	~~20~~	~~21~~	~~23~~	25	~~29~~	~~30~~	~~35~~	~~41~~
3rd roll (2nd roll – 6)	~~0~~		~~2~~	~~3~~	~~4~~				9		~~14~~			~~19~~		~~24~~		
4th roll (3rd roll + 11)			~~13~~	~~14~~	~~15~~				20							~~35~~		
5th roll (4th roll – 8)					~~7~~				12									

6. In each case, the number on the left decreases to the number on the right.
   However, it's not an inverse relationship, otherwise as the numbers on the left increase, the numbers generated would decrease.

It's clearly not a proportional (linear) relationship either. A very quick sketch XY graph (positive quadrant only) suggests a parabolic curve which hits the left-hand number axis at a number which is low, but greater than zero.



Try squaring the rh number and re-plotting



There definitely does seem to be a linear relationship between each left-hand number and the square of its right-hand number. Knowing that allows us to work out the **slope** or **gradient** of that line by using known coordinates of any two points on the line.

Gradient = (238 – 46) / (121 – 25) = 192 / 96 = 2.0

RH number	$(\text{RH number})^2$	2 X $(\text{RH number})^2$	LH number	LH number + 4
3	9	18	14	18
5	25	50	46	50
11	121	242	238	242
14	196	392	388	392

7. Both say "I was lying yesterday"

Day	Tweedledum	Tweedledee	Does that work?
Sunday	Tells the truth	Tells the truth	No
Monday	lies	Tells the truth	No
Tuesday	lies	Tells the truth	No
Wednesday	lies	Tells the truth	No
Thursday	Tells the truth	lies	**YES**
Friday	Tells the truth	lies	No

Saturday	Tells the truth	lies	No

8. It helps if you remember that
   4 is the sum of 1 and 3
   6 is the sum of 1, 2 and 3
   10 is the sum of 1, 2, 3, and 4
   15 is the sum of 1, 2, 3, 4 and 5
   16 is the sum of 1, 2, 3, 4 and 6
   21 is the sum of 1, 2, 3, 4, 5 and 6
   17 is the sum of 9 and 8
   16 is the sum of 9 and 7
   24 is the sum of 9, 8 and 7
   30 is the sum of 9, 8, 7 and 6
   35 is the sum of 9, 8, 7, 6 and 5

9. By inspection, it can be seen that the difference between the first row and the third row is that the third row has one red ball in place of the green ball in the first row. The third row has a total value of 47, while the first row has a value of 42. That must mean that the red ball has a value 5 greater than the green ball. Possibilities are: green =1, red = 6; green = 2, red = 7; green = 3, red = 8; green = 4, red = 9; and green = 5, red = 10.

   If red = 10 and green = 5, from row three, three blues plus one yellow =7; blue = 1 and yellow = 3 or blue = 2 and yellow = 1

   Try red = 10, green = 5, blue = 1 and yellow = 3 in row 2
   2 + 6 + 10 + 15 = 33 (INCORRECT)
   Try red = 10, green = 5, blue = 2 and yellow = 1 in row 2
   4 + 2 + 10 + 15 = 31 (CORRECT)
   Try red = 10, green = 5, blue = 2 and yellow = 1 in row 4
   6 + 3 + 10 + 5 = 24 (INCORRECT)

   So red is **not** 10 and green is **not** 5.

   If red = 9, green = 4 from row three, three blues plus one yellow =11; blue = 1 and yellow= 8 or blue = 2 and yellow = 5 or blue = 3 and yellow = 2

   Try red = 9, green = 4, blue = 1 and yellow = 8 in row 2
   2 + 16 + 9 + 12 = 39 (INCORRECT)
   Try red = 9, green = 4, blue = 2 and yellow = 5 in row 2
   4 + 10 + 9 + 12 = 35 (INCORRECT)
   Try red = 9, green = 4, blue = 3 and yellow = 2 in row 2
   6 + 4 + 9 + 12 = 31 (CORRECT)
   Try red = 9, green = 4, blue = 3 and yellow = 2 in row 4
   9 + 6 + 9 + 4 = 28 (INCORRECT)

   So red is **not** 9 and green is **not** 4.

   If red = 8, green = 3 from row three, three blues plus one yellow =15; blue = 2 and yellow= 9 is the only possibility

   Try red = 8, green = 3, blue = 2 and yellow = 9 in row 2
   4 + 18 + 8 + 9 = 39 (INCORRECT)

   So red is **not** 8 and green is **not** 3.

If red = 7, green = 2 from row three, three blues plus one yellow =19; blue = 5 and yellow= 4 OR blue = 6 and yellow = 1

Try red = 7, green = 3, blue = 5 and yellow = 4 in row 2
10 + 8 + 7 + 6 = 31 (CORRECT)
Try red = 7, green = 3, blue = 5 and yellow = 4 in row 4
15 + 12 + 7 + 2 = 36 (CORRECT)
Try red = 7, green = 3, blue = 5 and yellow = 4 in row 1
15 + 4 + 21 + 2 = 32 (CORRECT)

Solution is red = 7, green = 3, blue = 5 and yellow = 4

This can also be solved using simultaneous equations.
Let the value of each blue ball = ***b***
Let the value of each yellow ball = ***y***
Let the value of each red ball = ***r***
Let the value of each green ball = ***g***

$3b + y + 3r + g = 42$ {equation 1}

$2b + 2y + r + 3g = 31$ {equation 2}

$3b + y + 4r = 47$ {equation 3}

$3b + 3y + r + g = 36$ {equation 4}

Subtracting equation 1 from equation 3

$r - g = 5$ or $r = g + 5$

Substituting $(g + 5)$ for r in all equations gives

$3b + y + 4g = 27$ {equation 5}

$2b + 2y + 4g = 26$ (divide through by 2)
$b + y + 2g = 13$ {equation 6}

$3b + 3y + 2g = 31$ {equation 7}

Multiply equation 6 by 3

$3b + 3y + 6g = 39$ {equation 8}

Subtract equation 7 from equation 8
$4g = 8$

$g = 2$

$r - g = 5$

$r = 7$

Substituting $g = 2$ and $r = 7$ into equations 3 and 4

$3b + y + = 19$ {equation 9}

$3b + 3y = 27$ (divide through by 3)
$b + y = 9$ {equation 10}

Subtract equation 10 from equation 9

$2b = 10$

b = 5 thus from equation 10, ***y*** = 4

10. The Duke's answer bamboozled the Lord Chamberlain because if it's true, he'd be beheaded, which would make the statement false. If it's a lie, he'd be hanged, which would make it true. Since the Lord Chamberlain had no clear answer, he had to let the Duke go.