VIC SKEPTICS

Logic and Maths Puzzles 106 May 2021

1. BEST BEFORE ....

Four products at a local supermarket were marked down to clear because of imminent "Best Before" dates (BBDs) within the same week. For each product, using the clues and the grid supplied, can you provide the brand, the place of origin (including Victoria), the type of product, the flavour added to the product, and its BBD?

	2nd	3rd	5th	6th	Vic	NSW	WA	NZ	*Chicken*	*Bacon*	*Onion*	*Beef*	Biscuits	Chips	Dip	Sauce
REX																
BESTS																
QUEEN																
WHITE'S																
Biscuits																
Chips																
Dip																
Sauce																
Chicken																
Bacon																
Onion																
Beef																
Vic																
NSW																
WA																
NZ																

(i). The four products were the one bearing the *Rex* brand, the one labelled "Best Before 5th May", the dip, and the one produced in Western Australia.

(ii). The bacon-flavoured product has a later BBD than the beef-flavoured product.

(iii). The earliest BBD was borne by the chicken-flavoured product.

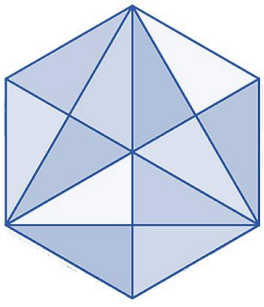
(iv). The onion-flavoured product from New South Wales was not Bests brand.

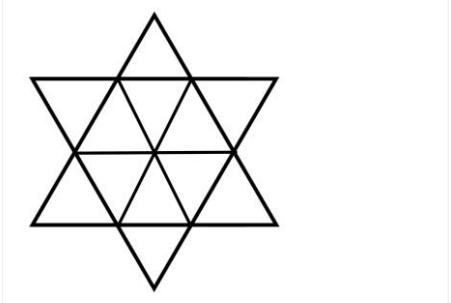
(v). The Queen potato chips' BBD was earlier than 6th May but later than 2nd May.

(vi). The White's product's BBD is later than the Best's product, which was not beef-flavoured or a product of New Zealand.

(vii). The dry biscuits' BBD is earlier than that of the potato chips and also earlier than that of the Bests product.

2. How many Triangles?





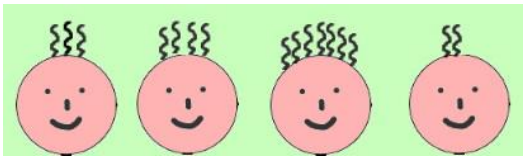
What is the total number of triangles in each figure?

3. PLATE NUMBERS

At the party, the hostess decided to serve food in this pattern... For every three guests, she would serve one plate of mixed nuts. For every five guests, she would serve one plate of cocktail sausages and one plate of party pies; and for every six guests she would serve one plate of chicken wings. At the end of the party, she had served a total of eighty-one plates. How many guests were at the party?

4. Baldly Going…

You have teleported down to a town in a hitherto unvisited planet, in which you discover the following:



i. No two inhabitants have the same number of hairs on their head.
ii. No inhabitant has exactly 4 hairs.
iii. There are more inhabitants in town than hairs on any individual inhabitant's head.
iv. None of the inhabitants is totally bald.

What is the highest possible number of inhabitants?

5. Figure It Out

If a man drives his car from his home to his work-place at a speed of 40km/h, he arrives 20 minutes late. If he travels at 60 km/h, he arrives 15 minutes early.
If he travels at a speed of 50 km/h, does he arrive:

exactly on time
5 minutes early
2 minutes early
1 minute early
1 minute late
2 minutes late
5 minutes late

One of the above answers is correct. Choose that answer and write it down.

6. Figure It Out

A set of football matches is to be organized in a "round robin" fashion; i.e. every participating teams plays a match against every other team once. If 55 matches are played, how many teams participated?

7.  Figure It Out

A solo dice game is played where, on each turn, a normal pair of dice is rolled. The score is calculated by taking the product, rather than the sum, of the two numbers shown on the dice.

In a particular game, the score for the second roll is five more than the score for the first; the score for the third roll is six less than that of the second; the score for the fourth roll is eleven more than that of the third; and the score for the fifth roll is eight less than that of the fourth.

a. What was the score for the first throw?
b. What was the score for the second throw?
c. What was the score for the third throw?
d. What was the score for the fourth throw?
e. What was the score for the fifth throw?

(Hint: for one throw, the lowest possible score is 1 X 1 =**1**; the highest possible score is 6 X 6 = **36**; what other scores are possible?)

8.  Figure It Out

Three sisters are identical triplets.
Sarah always tells the truth.
Sue always lies.
Sally sometimes lies and sometimes tells the truth.

Victor tried to identify them by asking each of them one question.

He asked the sister that was sitting on the left, "Which sister is in the middle?" and the answer he received was, "That's Sarah."

He then asked the sister in the middle, "What is your name?" The response was, "I'm Sally."

Victor then asked the sister on the right, "Who is that in the middle?" The sister then replied, "She is Sue."

This confused Victor; he had asked the same question three times and received three different answers.

a. Who was on the left?
b. Who was in the middle?
c. Who was on the right?

9. Petty Stock-take

At a Pet Show recently I noticed that all except eight of the entries were cats, all except eight were dogs, all except eight were canaries, all except eight were hamsters and all except eight were goldfish.

How many of each animal were at the Pet Show?

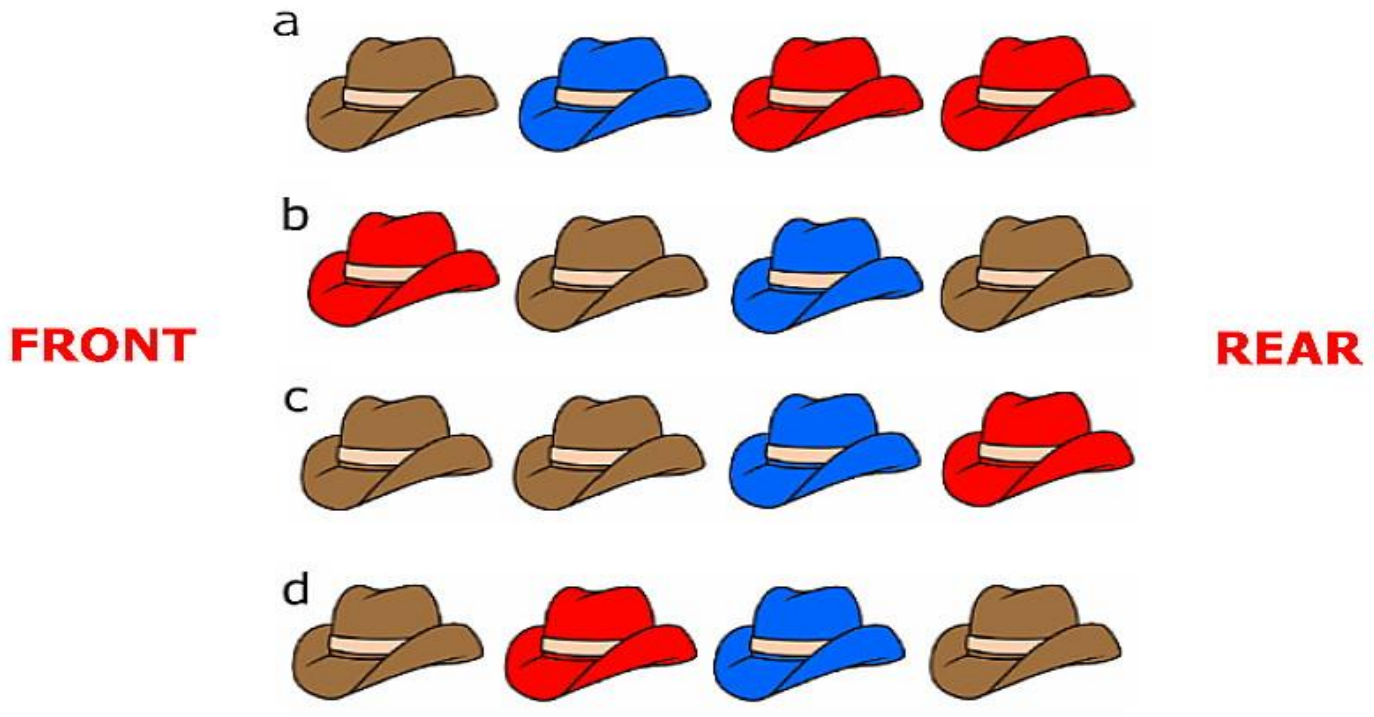
10. # HAT's INTERESTING...

A teacher lined up four students so that each one could see the student in front of him but not behind him. Each had a hat placed on his head; so the student at the back could see the hats of the three students in front, but the student in front could not see any hats.

"*There is a brown hat, a red hat, a blue hat, and a hat that is a duplicate of one of those colors,*" the teacher said.

Starting with the one in the back, each student was asked what color hat they were wearing. They all gave the correct answer!

Which one of these arrangements would make this possible?



Answers next page

Answers:

Worked solutions follow the answers.

1. Best Before Date 2nd May – REX Brand Chicken-flavoured biscuits from Victoria.
   Best Before Date 3rd May – BESTS Brand bacon-flavoured sauce from WA.
   Best Before Date 5th May – QUEEN Brand beef-flavoured chips from NZ.
   Best Before Date 6th May – WHITES Brand onion-flavoured dip from NSW.

2. (a) 37 triangles (b) 20 triangles

3. 90 guests

4. 3 inhabitants

5. 1 minute early

6. 11 teams

7. a. 10 b. 15 c. 9 d. 20 e. 12

8. a. (left): Sally b. (Middle): Sue c. (Right): Sarah

9. two dogs, two cats, two canaries, two hamsters and two fish

10. Arrangement C

SOLUTIONS:

1. Let's start by recording all the direct information from the clues.

	2nd	3rd	5th	6th	Vic	NSW	WA	NZ	*Chicken*	*Bacon*	*Onion*	*Beef*	Biscuits	Chips	Dip	Sauce
REX			X				X							X	X	
BESTS	X			X		X		X			X	X	X	X		
QUEEN	X			X									X	✓	X	X
WHITE'S	X													X		
Biscuits			X	X												
Chips	X			X												
Dip			X				X									
Sauce																
Chicken	✓	X	X	X		X										
Bacon	X					X										
Onion	X				X	✓	X	X								
Beef	X			X		X										
Vic																
NSW																
WA			X													
NZ																

REX brand remains the only candidate for the 2nd May BBD and is therefore also the one with the chicken flavour. Logically, since the REX product, the one with the 2nd May BBD and the chicken-flavoured item are the same product, they share the same place of origin (not NSW, WA or NZ – hence Victoria) and are the same type of food (biscuits or sauce, but not chips or dip) The grid can now be added to:

	2nd	3rd	5th	6th	Vic	NSW	WA	NZ	*Chicken*	*Bacon*	*Onion*	*Beef*	Biscuits	Chips	Dip	Sauce
REX	✓	X	X	X	✓	X	X	X	✓	X	X	X		X	X	
BESTS	X			X	X	X		X	X		X	X	X	X		
QUEEN	X			X	X				X				X	✓	X	X
WHITE'S	X				X				X					X		
Biscuits			X	X												
Chips	X			X	X				X							
Dip	X		X		X		X		X							
Sauce																
Chicken	✓	X	X	X	✓	X	X	X								
Bacon	X				X	X										
Onion	X				X	✓	X	X								
Beef	X			X	X	X										
Vic	✓	X	X	X												
NSW	X															
WA	X		X													
NZ	X															

It can be seen that BESTS brand is the only candidate remaining for WA as place of origin, and has a bacon flavour (not chicken, onion or beef)

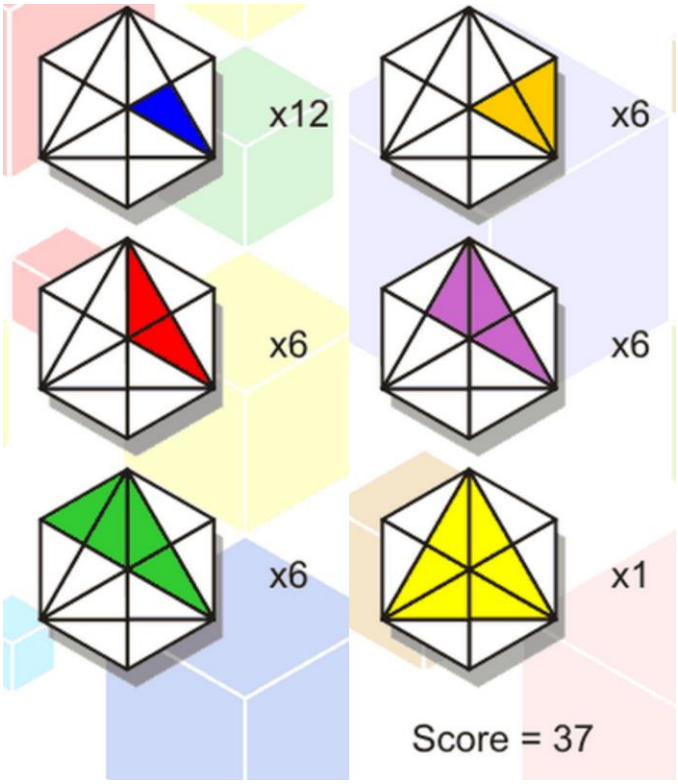
It can also be seen that WHITES brand is the only candidate for a BBD of 6th May.

The grid can now be completed.

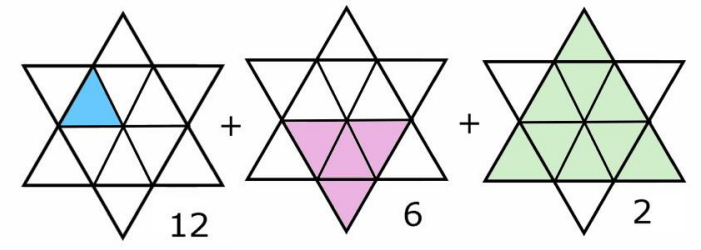
	2nd	3rd	5th	6th	Vic	NSW	WA	NZ	*Chicken*	*Bacon*	*Onion*	*Beef*	Biscuits	Chips	Dip	Sauce
REX	✓	X	X	X	✓	X	X	X	✓	X	X	X	✓	X	X	X
BESTS	X	✓	X	X	X	X	✓	X	X	✓	X	X	X	X	X	✓
QUEEN	X	X	✓	X	X	X	X	✓	X	X	X	✓	X	✓	X	X
WHITE'S	X	X	X	✓	X	✓	X	X	X	X	✓	X	X	X	✓	X
Biscuits	✓	X	X	X	✓	X	X	X	✓	X	X	X				
Chips	X	X	✓	X	X	X	X	✓	X	X	X	✓				
Dip	X	X	X	✓	X	✓	X	X	X	X	✓	X				
Sauce	X	✓	X	X	X	X	✓	X	X	✓	X	X				
Chicken	✓	X	X	X	✓	X	X	X								
Bacon	X	✓	X	X	X	X	✓	X								
Onion	X	X	X	✓	X	✓	X	X								
Beef	X	X	✓	X	X	X	X	✓								
Vic	✓	X	X	X												
NSW	X	X	X	✓												
WA	X	✓	X	X												
NZ	X	X	✓	X												

2. (a) How Many triangles? Solution

The shaded portion in each case shows one representative triangle of a given shape and size; the adjacent number represents the number of times such a triangle recurs in the diagram.



(b) How Many triangles? Solution



Score = 20

3. Assuming the plates were served to exact whole numbers, the number of guests must have been a common multiple of 3, 5 and 6.
Possibilities are 30, 60, 90, 120 etc. Trial - and- error is one approach:

Number of guests	Plates of Mixed nuts	Plates of Cocktail sausages	Plates of party pies	Plates of chicken wings	Total plates
30	10	6	6	5	27
60	20	12	12	10	54
90	30	18	18	15	81

Alternative algebraic solution:

Let the number of guests be ***n***

Then the number of plates of mixed nuts = ***n***/3

The number of plates of cocktail sausages = ***n***/5

(continued next page)

The number of plates of party pies = $n/5$
The number of plates of chicken wings = $n/6$

$n/3 + 2n/5 + n/6 = 81$

{apply a lowest common factor of 30}

$$\frac{10n + 12n + 5n}{30} = 81$$

$27n = 81 \times 30$

$n = 9$

4. Could there be only one inhabitant?

The rules are:

i. No two inhabitants have the same number of hairs on their head. (not applicable)

ii. No inhabitant has exactly 4 hairs. (Would allow for the one inhabitant to have any number of hairs but 4.)

iii. There are more inhabitants in town than hairs on any individual inhabitant's head.

iv. None of the inhabitants is totally bald. (**Taking these two rules together eliminates the possibility of having only one inhabitant.)**

Could there be two inhabitants?

ii. No inhabitant has exactly 4 hairs.

iv. None of the inhabitants is totally bald. (Allows for any number of hairs except 0 or 4)

i. No two inhabitants have the same number of hairs on their head.

iii. There are more inhabitants in town than hairs on any individual inhabitant's head.

(If there are only two inhabitants, one of them must have one hair, the other one being bald. It's **not possible** to have only two inhabitants.

Could there be three inhabitants? Yes. Applying all the rules, you could have three inhabitants, one with one hair, the other with two hairs.

Could there be four inhabitants?

i. No two inhabitants have the same number of hairs on their head.

ii. ii. No inhabitant has exactly 4 hairs.

iv. None of the inhabitants is totally bald. (Taking these three rules alone would allow for any number of hairs other than 0 or 4, so long as the number was not duplicated.)

iii. There are more inhabitants in town than hairs on any individual inhabitant's head.

(If there are four inhabitants, the possible number of hairs on each head is 0, 1, 2 and 3 respectively. One of them is bald. **Not possible.**

Could there be five inhabitants?

i. No two inhabitants have the same number of hairs on their head.

ii. No inhabitant has exactly 4 hairs.

iv. None of the inhabitants is totally bald. (Taking these three rules alone would allow for any number of hairs other than 0 or 4, so long as the number was not duplicated.)

iii. There are more inhabitants in town than hairs on any individual inhabitant's head.

(If there are five inhabitants, the possible number of hairs on each head is 1, 2 or 3. This is **not possible** without breaking rule 1.)

If you follow this logic through it reveals that the only correct answer is three inhabitants.

In fact, you can find versions of this puzzle on the internet with much larger numbers. (For some reason, clue ii often reads *No inhabitant has exactly 518 hairs*, with clues i, iii and iv unchanged. (The number 518 is quite arbitrary). The effect of this is to make the answer 517 inhabitants.

5. Let the distance driven be ***d*** kilometres, and ***t*** hours the time taken to arrive exactly on time.
   Then if speed equals distance travelled divided by time taken
   When he travels at 40 km/h and is 20 minutes late:

   $40 = \boldsymbol{d}/(\boldsymbol{t} + 1/3)$ or $\boldsymbol{d} = 40\boldsymbol{t} + 40/3$ {equation 1}

   When he travels at 60 km/h and is 15 minutes early:

   $60 = \boldsymbol{d}/(\boldsymbol{t} - 1/4)$ or $\boldsymbol{d} = 60\boldsymbol{t} - 60/4$ {equation 2}

   $40\boldsymbol{t} + 13\frac{1}{3} = 60\boldsymbol{t} - 15$

   $20\,\boldsymbol{t} = 28\frac{1}{3} = \frac{85}{3}$

   $\boldsymbol{t} = \frac{85}{60}$ {optimum travel time is 1 hr 25 min}

   {substituting in equation 2}

   $\boldsymbol{d} = 60\boldsymbol{t} - 15$

   $\boldsymbol{d} = 60 \times \frac{85}{60} - 15 = 70$ {distance is 70 km}

   travelling 70 km at 50 km/h

   $\boldsymbol{t} = \frac{70}{50} = 1.4$ hours = 1 hour 24 minutes which is 1 min early.

6. If there are n teams, each team needs to play (n-1) games to complete a full round, where each team has played each other team once. However, Team A playing Team B is the same event as Team B playing team A. So the number of actual games in each round is not n(n-1), it's n(n-1)/2.

   $n(n-1)/2 = 55$

   $n(n-1) = 110$

   $n^2 - n = 110$

   $n^2 - n - 110 = 0$ {factorise)

   $(n + 10)(n - 11) = 0$

   $n = -10$ or $+ 11$ as n must be a positive whole number, n = 11

7. Possible scores are: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30 and 36
"the score for the second roll is five more than the score for the first"
Possibilities are:

First roll	Second roll
1	6
3	8
4	9
5	10
10	15
15	20
25	30

"the score for the third roll is six less than that of the second"

second roll	Third roll	Possible?
6	0	no
8	2	yes
9	3	yes
10	4	yes
15	9	yes
20	14	no
30	24	yes

"the score for the fourth roll is eleven more than that of the third"

Third roll	fourth roll	Possible?
2	13	no
3	14	no
4	15	yes
9	20	yes
24	35	no

"and the score for the fifth roll is eight less than that of the fourth"

fourth roll	fifth roll	Possible?
15	7	no
20	12	yes

8. Start with all possibilities:

LEFT	MIDDLE	RIGHT
Sarah, Sue, Sally	Sarah, Sue, Sally	Sarah, Sue, Sally

He asked the sister that was sitting on the left, "Which sister is in the middle?" and the answer he received was, "That's Sarah."
Sarah always tells the truth. So the girl sitting on the left can't have been Sarah
The girl on the left may have been Sue, who always lies, in which case the girl in the middle was Sue or Sally.
The girl on the left may have been Sally, who sometimes lies, in which case the girl in the middle could have been Sarah, Sue or Sally.

LEFT	MIDDLE	RIGHT
If Sue, (Lying)	Sally	Sarah
If Sally, (telling the truth)	Sarah	Sue
If Sally, (lying)	Sue	Sarah

He then asked the sister in the middle, "What is your name?" The response was, "I'm Sally."
If it was Sally, telling the truth, then the correct order left to right is Sue, Sally, Sarah
It can't have been Sarah. (Sarah is on the right)
If it was Sue, then the correct order left to right is Sally, Sue, Sarah

LEFT	MIDDLE	RIGHT
If Sue, (Lying)	Sally	Sarah
If Sally, (lying)	Sue	Sarah

Victor then asked the sister on the right, "Who is that in the middle?" The sister then replied, "She is Sue."
As the girl on the right must be Sarah, this statement must be true.

LEFT	MIDDLE	RIGHT
Sally (lying)	Sue	Sarah

9. From the text:
There are five kinds of animal
There is the same number of each kind of animal
4 kinds of animals represents eight individuals in each example
Hence there are two of each kind of animal or 5 X 2 = 10 animals in total

10. The only arrangement in which the person at the rear can see that his colour is missing from the three hats in front of him is (c). He can conclude that he has a red hat. The person in third-last place can then see two brown hats in front of him. He knows that there must be at least one red hat and one blue hat, and if the last in line is wearing the red hat, he must be wearing the blue. The person in first place and second place from the front can conclude that if the rear-most and second rear-most people were confident in their selections, it can only have been because they knew that there was only one hat of their colour (red and blue respectively). The brown hat is the one which is duplicated.