SPECIAL July 2020 – PUZZLES No 28 Logic & Maths (With Solutions)

1. When sorting the mail, Postman Pete found three parcels without labels, and the three labels that had fallen off the parcels. He had no idea which label belonged to which parcel, so he decided to take a guess, and stuck on label back on each parcel at random.
What was the probability that he got
(a) Every parcel incorrectly labeled?
(b) only one parcel correctly labeled?
(c) only two parcels correctly labeled?
(d) all three parcels correctly labeled?

2. It would take Andy four hours to paint a certain fence on his own. Barry would be able to paint the same fence in five hours, while Claude would require six hours. How long would it take them to paint that fence, to the nearest minute, if they were working together on the job?
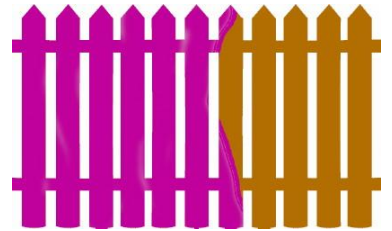
3. If two hours ago it was as long after one o'clock in the afternoon as it was before one o'clock in the morning, what time would it be now?
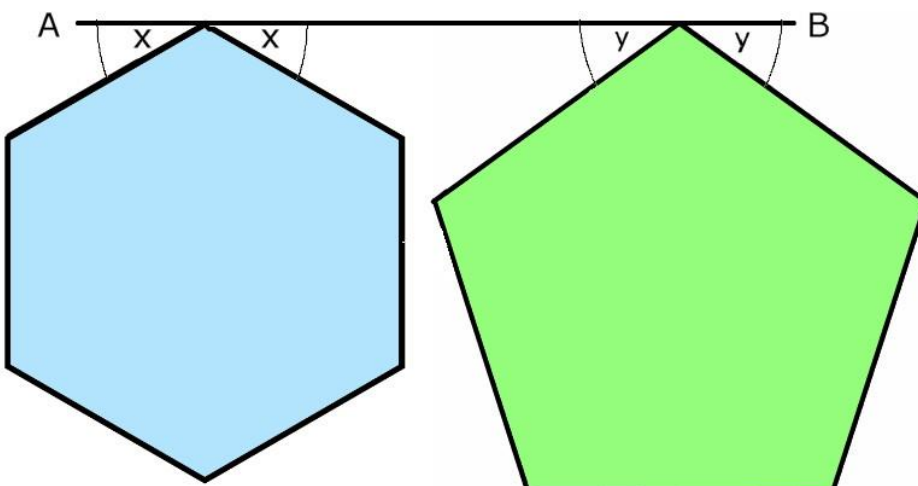
4. Alice, Ben, Charlie, David and Ed entered a contest to guess how many jelly-beans there were in a jar. Alice guessed 30, Ben guessed 28, Charlie guessed 29, David guessed 25 and Ed guessed 26.

Two of them were out by 1. One was wrong by 4, and another was wrong by 3.

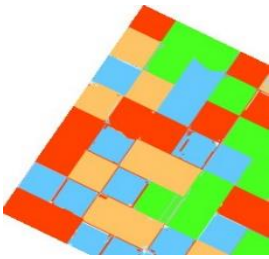One of them was correct. Who was it, and how many jelly-beans were in the jar?

5.

If AB is a straight line,

the blue figure is a regular hexagon

and the green figure is a regular pentagon:
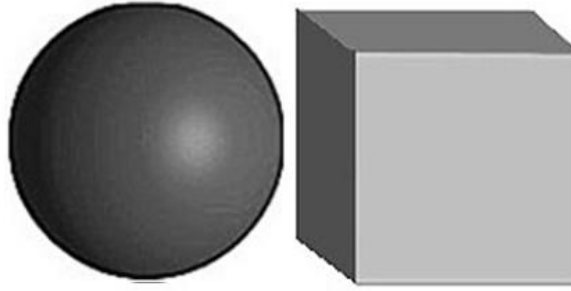
a. What is the size of angle x in degrees?

b. What is the size of angle y in degrees?

6. Kirsten makes a mosaic pattern from coloured tiles. One sixth of her tiles are blue, three eighths are orange, one quarter are green and the other 20 are red.

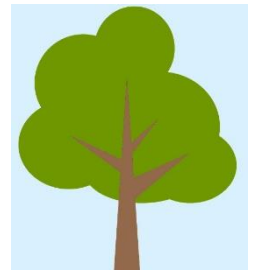How many tiles make up Kirsten's mosaic?

7. Which has the greater volume? A 5m wide sphere, or a 4m wide cube? (Drawn here to scale)
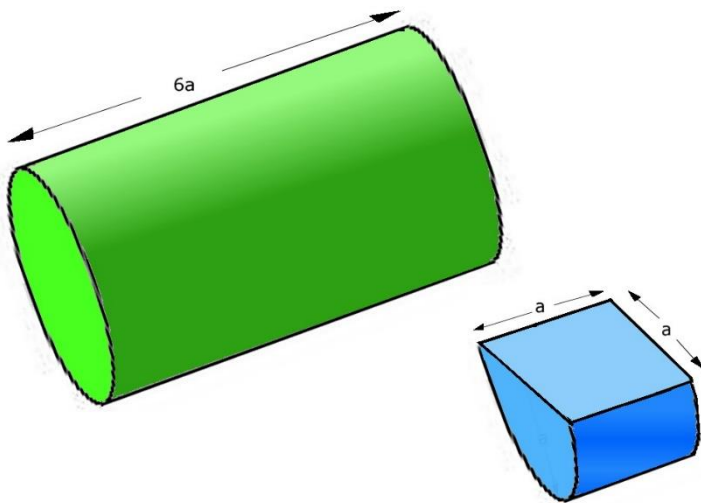
8. What percentage is 3% of 3%?

9. A shrub 1m in height when planted on the first of January 2000 as a seedling increased its height by 20% every year.

In what month of what year did it first exceed 3 m in height?

10.

The green figure represents a cylinder of radius a units and length $6a$ units

The blue figure represents a semi-circular prism of length and width both a units and is NOT drawn to scale in comparison to the cylinder.

How many <u>times</u> greater in volume is the green cylinder compared to the blue semi-circular prism?

6a

a

a

Solutions:

1. (a) Every parcel incorrectly labelled? 2 in 6 or 1 in 3 or 2 to 1

(b) only one parcel correctly labelled? Evens, 3 in 6 or 1 to 1 or 50%

(c) only two parcels correctly labelled? Zero

(d) all three parcels correctly labelled? 1 in 6 or 5 to 1

Let's call the parcels A, B, C and the labels X, Y and Z. Assume the correct match is parcel A with label X, parcel B with Label Y and parcel C with label Z


However, that table is misleading because it suggests that there are nine equally likely outcomes. In fact there are only **six**, because once a label is assigned to the first parcel, there are only two parcels and two labels left; choice of assigning parcels to labels has been limited. For example, it's not possible to label only two parcels out of three correctly. If the first two parcels are labelled correctly, so must the third.

Parcel A labelled X. then Parcel B labelled Y and Parcel C labelled Z - All correct.
Parcel A labelled X. then Parcel B labelled Z and Parcel C labelled Y - one correct
Parcel A labelled Y. then Parcel B labelled X and Parcel C labelled Z - one correct
Parcel A labelled Y. then Parcel B labelled Z and Parcel C labelled X – all incorrect
Parcel A labelled Z. then Parcel B labelled X and Parcel C labelled Y – all incorrect
Parcel A labelled Z. then Parcel B labelled Y and Parcel C labelled X – one correct

(This can also be represented as a tree diagram – it gives a better idea of how each choice limits later choices)
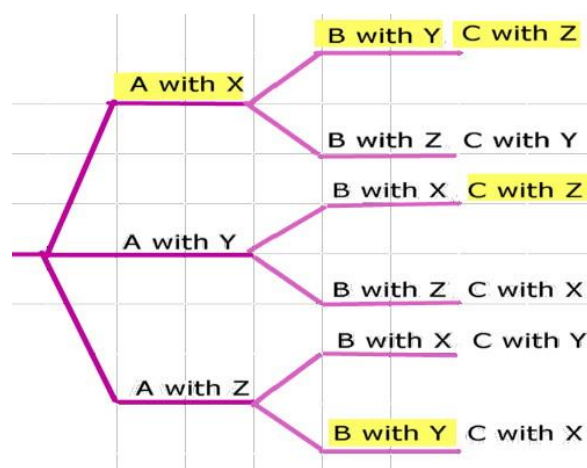
2. 1 hour 37 minutes.

If Andy requires four hours to paint the fence then his RATE of fence painting is $\frac{1}{4}$ of the fence per hour. Barry's is $\frac{1}{5}$ and Claude's is $\frac{1}{6}$.

You need to assume that when all three are painting together, the rate at which the fence is being painted is the <u>sum</u> of their three rates.

$\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ [Lowest common denominator is 60]

$= \frac{15+12+10}{60} = \frac{37}{60}$ (think of it as "fences per hour")

The whole job is therefore completed in $\frac{60}{37}$ hours (think of it as "hours per fence")

Which is $\frac{60 \, X \, 60}{37} = \frac{3600}{37}$ = 97.297 minutes = approx. 1 hour 37 minutes

3. Nine O'Clock (in the evening) or 9 pm

4. Charlie guessed right with 29

By process of elimination, Alice's guess of 30 can't be right because it requires a "wrong guess" of 27 or 33. There are no guesses of either 27 or 33.
Ben guessed 28. That can't be right because there needs to be a "wrong answer" of either 24 or 32. There are no guesses of either 24 or 32.
If Charlie's guess of 29 is correct, it agrees with two "wrong guesses" that are 1 out (28 and 30), one wrong by 4 (25) and one wrong by 3 (26). Charlie could be the right answer.
David's guess of 25 compiles with only one "out by 1"; there must be two. Even though his guess agrees with the other clues, David is eliminated.
Ed's guess of 26 is eliminated because only one other guess (25) is "out by 1".

That leaves only Charlie's guess of 29 as the correct guess.

5. Angle x is 30°   angle y is 36°

The key to this is that the two polygons are regular. That means their internal angles are equal and can be calculated using the internal angle of a polygon formula.

$$\theta = \frac{(n-2) \, X \, 180}{n}$$ where θ is the internal angle and n is the number of sides.

The blue figure to the left is a regular hexagon

$$\theta = \frac{(6-2) \, X \, 180}{6}$$

= 720/6 = 120°

as the three angles x, x and 120^0 are on a straight line, $x + x + 120 = 180$

$x = 30$

The green figure to the left is a regular pentagon.

$$\theta = \frac{(5 - 2) \ X \ 180}{5}$$
$= 540/5 = 108^0$

as the three angles form a straight line, $y + y + 108 = 180$

$y = 36$

6. 96 tiles

Let the total number of tiles be n.

$\dfrac{n}{6} + \dfrac{3n}{8} + \dfrac{n}{4} + 20 = n$ [lowest common denominator = 24]

$\dfrac{4n+9n+6n}{24} + 20 = n$

$\dfrac{19n}{24} + 20 = n$

$n - \dfrac{19n}{24} = 20$

$\dfrac{24n-19n}{24} = 20$

$\dfrac{5n}{24} = 20$

$5n = 480$

$n = 96$

7. The sphere (65.4 m^3 compared with cube 64 m^3)

Calculated using the formulas (formulae?) for the Volume of a sphere V=4/3Πr^3 where r = 2.5
and Volume of a cube = l^3 where l = 4.

8. 0.09%

$\dfrac{3}{100} \ X \ \dfrac{3}{100} = \dfrac{9}{1000}$

9. January 2006

The way the question is written implies that the shrub's height will increase each year by 20% of its height **at the end of the previous year**. In other words, it's a COMPOUNDING calculation, like compound interest on a purchase. Probably the simplest way to tackle this is to multiply the height on January 1 of each year by 1.2 to calculate the height on January 1 the following year. This can done simply using a calculator with 1.2 in the memory, starting with 1.0 for January 1 2000 and successively multiplying by 1.2 ("recall memory"). The result after 6 successive multiplications

$1.0 \times 1.2 \times 1.2 \times 1.2 \times 1.2 \times 1.2 \times 1.2$ = approximately 2.98 metres

In other words, the tree is 2.98 metres tall on January 1 2006 and at the rate of growth will exceed 3.0 metres early in January 2006.

(In case you're worried about this being a trick question with periods of dormancy during winter – January in Australia is high summer.)

10. **48** times greater

The calculation for the volume of a cylinder is given by $V = \prod r^2 h$ where r is the cylinder's radius and h is its length

As the calculation of the volumes of the two shapes will have several common terms, which will cancel out when we divide one into the other, we won't need to calculate out Volumes completely.

Green cylinder: $V = \prod r^2 h = \prod \times a^2 \times 6a = 6 \prod a^3$

Blue semi-circular prism: note that the radius is $\dfrac{a}{2}$, not a.

$$V = \frac{\prod r^2 h}{2} = \frac{\prod \times a^2 \times a}{2 \times (2 \times 2)} = \frac{\prod a^3}{8}$$

Volume of Green Cylinder ÷ Volume of blue semi-circular prism

$$= (6\cancel{\prod}a^3) \div \frac{\cancel{\prod}a^3}{8}$$

$$= 6 \times 8 = 48$$

The green cylinder is 48 times the volume of the blue semi-circular prism