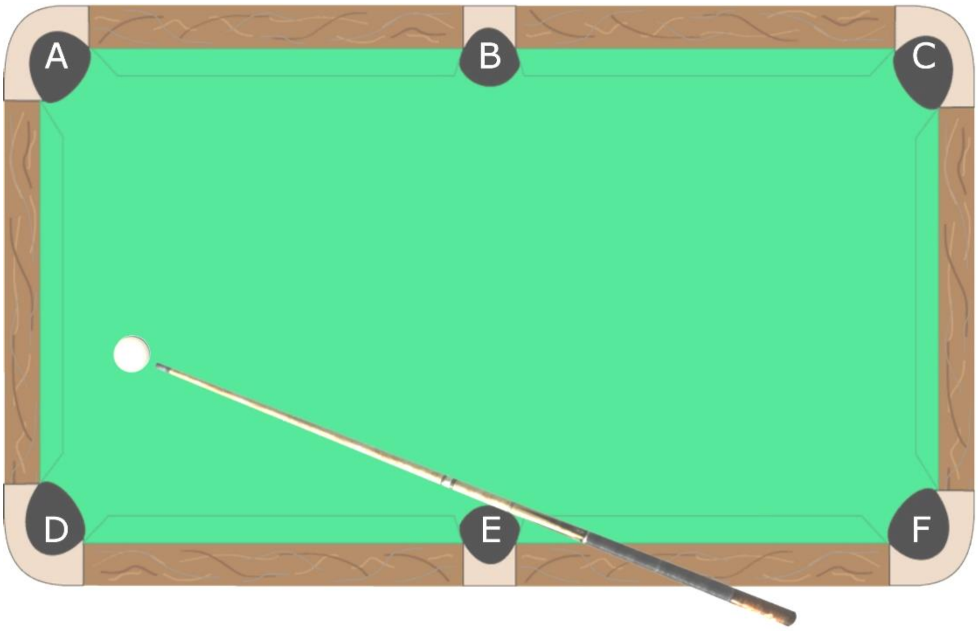
Logic and Maths Puzzles # 73 August 2018

1.



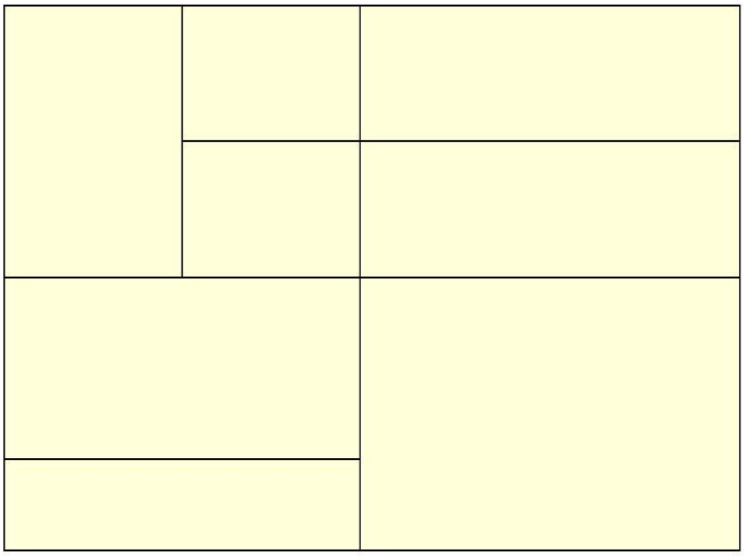
Assuming the cue ball is struck cleanly and with the correct weight, and the cushions play true, which pocket will the cue ball end up in?

2. **2!** is called 2 shriek or 2 factorial and is **2 X 1 =2**

3! is called 3 shriek or 3 factorial and is **3 X 2 X 1 = 6**

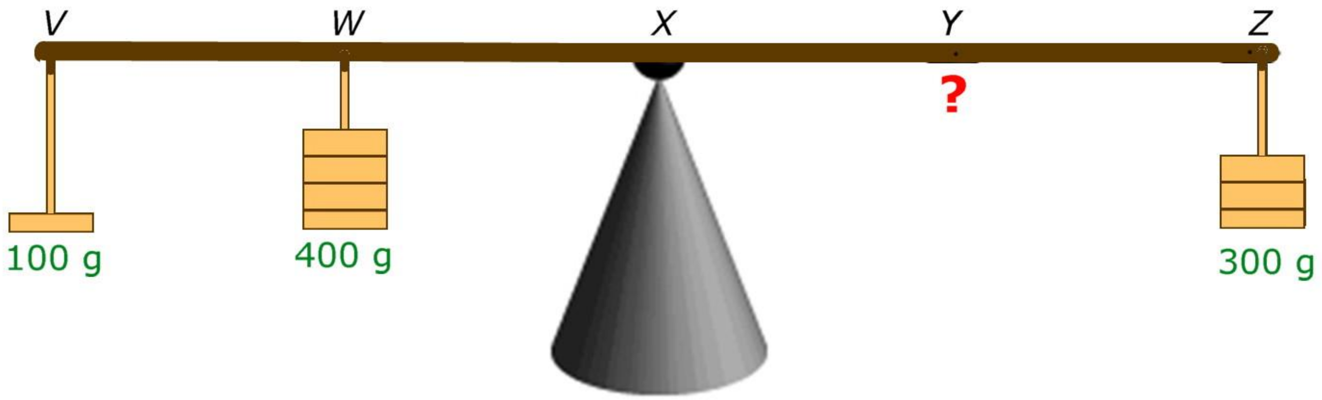
What is the value of the **product** of **4!** and **5!** ?

3.



How many rectangles are in this figure in total?

4.



A bar is marked at points V, W, X, Y and Z.
The distances between V and W, between W and X, between X and Y and between Y and Z are equal.

The empty bar (without any masses hanging from it) balances at point X.

When masses are hung from the bar as shown in the diagram, what amount of mass in grams will be required to be hung at point Y for the bar to be restored to balance?

5. If I start with $2000 in a bank account, and the amount of money in this account decreases by 80% each year, how much money will be in the account after five years?

6. In a certain army, you only salute officers of equal or higher rank to yourself. Two Generals, two Colonels, two majors, two captains and two lieutenants enter a room.

How many salutes are there?

7. If 180 is the highest possible score a competitor can achieve in a round of darts, what are the next three highest possible scores?

8. What is the most likely score from:

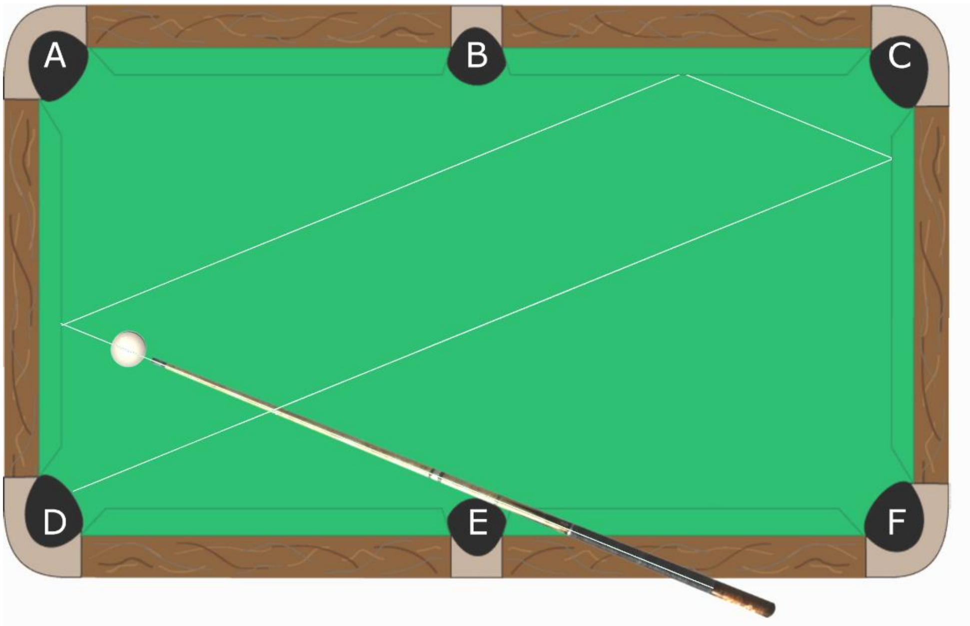
a. one throw of two dice?
b. one throw of 3 dice?
c. What are the two least likely scores from one throw of four dice?

9. What percentage is 9% of 9% of 90%?

10. What is the more common name for a regular quadrilateral?

Solutions:

1. pocket D

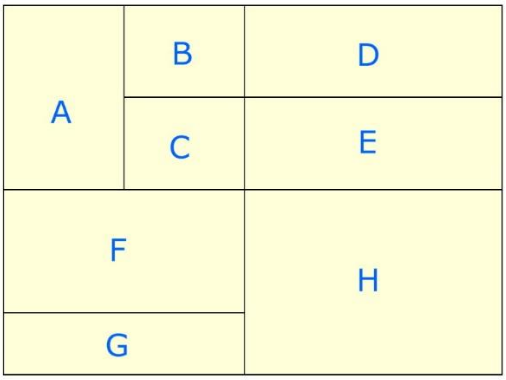


2. 2,880

A **product** of two or more numbers is the answer you get by multiplying them. The product of 2 and 7 is 14, for example

The product of 4! and 5! Is therefore
(4 X 3 X 2 X 1) X (5 X 4 X 3 X 2 X 1)
= 24 X 120
= 2,880

3. 22



The overall shape consists of 8 regions each consisting of a single rectangle. (A to H) Obviously other larger rectangles exist which are combinations of two or more of the lettered rectangles. For example, A + B + C is a rectangle.
Making sure you only count each such combination ONCE,

All rectangles containing **A:** There are 6 of them

A, **A + B + C**, **A + B + C + D + E**, **A + B + C + F**, **A + B + C + F + G**, and **A + B + C + D + E + F + G + H**

All rectangles containing **B**, but not **A**: There are 4 of them *(continued overpage)*

B, **B + C**, **B + D**, **B + C + D + E**

All rectangles containing **C**, but not **A** or **B**: There are 2 of them

C, **C + E**

All rectangles containing **D**, but not **A**, **B** or **C**: There are 3 of them

D, **D + E**, **D + E + H**

All rectangles containing **E**, but not **A**, **B**, **C** or **D**: There are 2 of them

E, **E + H**

All rectangles containing **F**, but not **A**, **B**, **C**, **D** or **E**: There are 3 of them

F, **F + G**, **F + G + H**

All rectangles containing **G**, but not **A**, **B**, **C**, **D**, **E** or **F**: There is 1 of them

G

All rectangles containing **H**, but not **A**, **B**, **C**, **D**, **E**, **F** or **G**: There is 1 of them

H

Total rectangles: 6 + 4 + 2 + 3 + 2 + 3 + 1 + 1 = **22**

4. **Zero grams.** The bar is already in balance

To solve this it is necessary to understand the principle of turning moments of forces around a pivoting point. In the diagram, masses hung to the left of the pivot point will tend to cause the bar to rotate anti-clockwise, because they exert a weight force due to gravity.
Masses hung to the right of the pivot point will tend to cause the bar to rotate clockwise.
The amount to which a mass tends to rotate the bar depends on two things:

- The size or magnitude of the mass
- The distance of the mass from the pivot point

The **turning moment** of a force is given by the force X the distance from the pivot point

If the bar is balanced, The SUM of turning moments of all the forces on the left hand side must equal the sum of all the turning moments on the right hand side.
Since the weight exerted by any mass is directly proportional to the size of the mass, it's convenient to regard each turning moment in the diagram as (mass) X (distance)

Let the distance between X and Y = d
Let the mass required to restore balance = m

The sum of the turning moments to the left of the pivot point are therefore
(400 X d) + (100 X 2d)
The sum of the turning moments to the right of the pivot point are
(m X d) + (300 X 2d)

If the bar is in balance,

$(400 \text{ X } d) + (100 \text{ X } 2d) = (m \text{ X } d) + (300 \text{ X } 2d)$

$400d + 200d = md + 600d$

$600d = md + 600d$

Dividing through by d

$600 = m + 600$

$m = 0$

5. 64 cents

The simplest way of looking at this problem is to realise that if 80% is lost from the account annually, 20% or one fifth remains, which becomes the starting balance for the next year.
Beginning of year 1: Balance is $2000
Beginning of year 2: Balance is one fifth of $2000 or $400
Beginning of year 3: Balance is one fifth of $400 or $80
Beginning of year 4: Balance is one fifth of $80 or $16
Beginning of year 5: Balance is one fifth of $16 or $3.20
End of year 5: Balance is one fifth of $3.20 or $0.64

6. 50 salutes

Lieutenant 1 salutes lieutenant 2, captain 1, captain 2, major 1, major 2, colonel 1, colonel 2, general 1 and general 2: That’s 9 salutes

Lieutenant 2 salutes lieutenant 1, captain 1, captain 2, major 1, major 2, colonel 1, colonel 2, general 1 and general 2: That’s 9 salutes

Captain 1 salutes captain 2, major 1, major 2, colonel 1, colonel 2, general 1 and general 2: That’s 7 salutes

Captain 2 salutes captain 1, major 1, major 2, colonel 1, colonel 2, general 1 and general 2: That’s 7 salutes

Major 1 salutes major 2, colonel 1, colonel 2, general 1 and general 2: That’s 5 salutes

Major 2 salutes major 1, colonel 1, colonel 2, general 1 and general 2: That’s 5 salutes

Colonel 1 salutes colonel 2, general 1 and general 2: That’s 3 salutes

Colonel 2 salutes colonel 1, general 1 and general 2: That’s 3 salutes

General 1 salutes general 2: that’s 1 salute

General 2 salutes general 1: that’s 1 salute

Total salutes = 9 + 9 + 7 + 7 + 5 + 5 + 3 + 3 + 1 + 1 = 50

7. 177, 174, 171

180 is achieved by 3 throws of treble 20.
177 is achieved by 2 throws of treble 20 and 1 throw of treble 19

174 is achieved by 1 throws of treble 20 and 2 throws of treble 19
171 is achieved by 3 throws of treble 19

8. a. 7 b. Scores of 10 and 11 are equally likely c. 4 and 24

		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

a. A score of 7 is thrown with a frequency of 6 in 36 or 1 in 6 throws

b.

Possible score	How made: (die 1, die 2, die 3)	ways
3	1,1,1	1
4	1,1,2 1,2,1 2,1,1	3
5	1,1,3, 1,3,1 3,1,1 2, 2,1 2,1,2 1,2,2	6
6	1,1,4 1,4,1 4,1,1 1,2,3 2,1,3 2,3,1 1,3,2, 3,1,2, 3,2,1	9
7		
8		
9		
10		
11		
12		
13		
14		
15	6,6,3 6,3,6 3,6,6 6,5,4, 6,4,5 4,6,5 5,6,4 4,5,6 5,4,6	9
16	6,6,4 6,4,6 4,6,6 6,5,5 5,6,5 5,5,6	6
17	6,6,5 6,5,6 5,6,6	3
18	6,6,6	1

From the above partly completed table, it can be seen that a pattern emerges from which the highest possible score for three dice can be predicted. It converges equally on both 10 and 11; those two scores are equally likely

With four dice, least likely outcomes are 1,1,1,1 producing a score of 4 and 6,6,6,6 producing a score of 24

9. 0.729%

9% of 9% of 90%

Expressed in decimal form

= 0.09 X 0.09 X 0.9

= 0.00729

Multiply by 100 to convert back to a %

= 0.729%

10. a square

“Quadrilateral” is a four-sided figure; i.e. a polygon in which the number of sides equals 4.
“Regular” means that all sides have the same length and that all internal angles are equal.