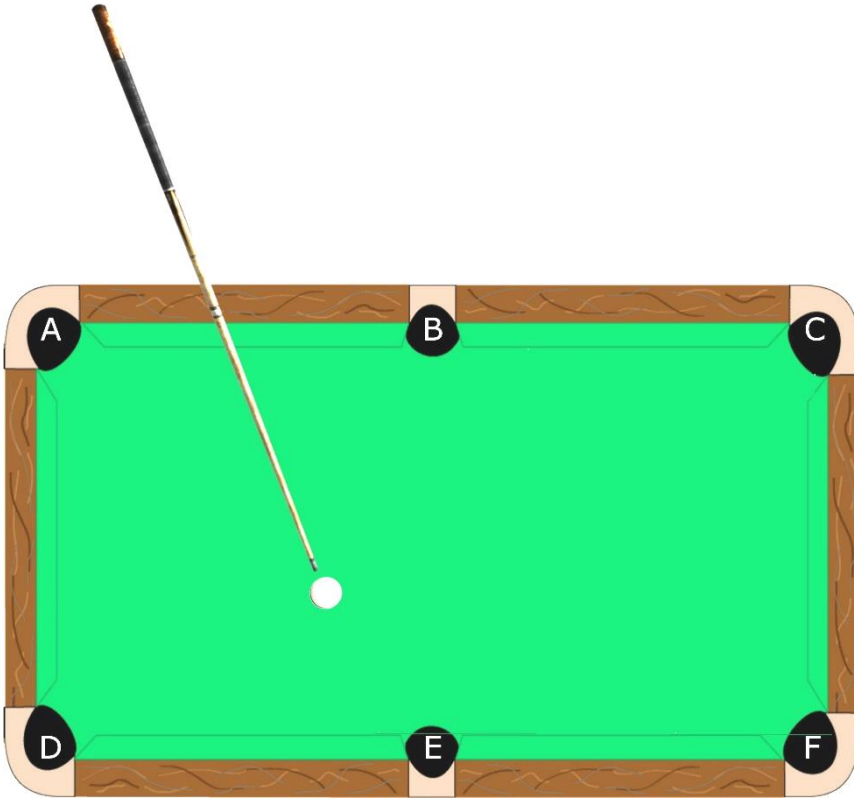# Logic and Maths Puzzles # 72 July 2018

1.

Assuming the cue ball is struck cleanly, the ball is struck with the correct weight and the cushions play true, which pocket will the cue ball end up in?
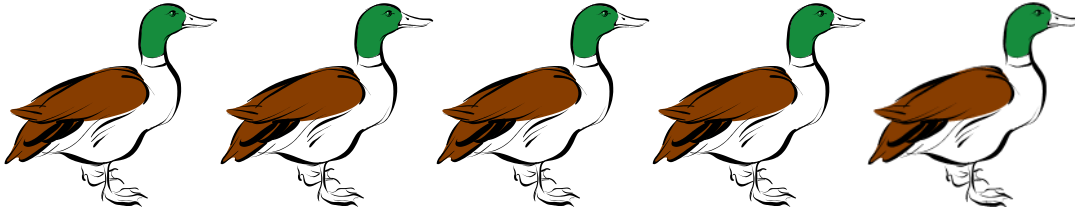
2.

The picture shows seven cards dealt face-up. Which cards were dealt first, third and fifth?

3. Is 157 a PRIME NUMBER?

4. What's three to the power of two minus two to the power of three?

5.

What is the minimum number of ducks that suits this description?

"Two ducks in front of two other ducks, two ducks behind two other ducks and two ducks beside two other ducks"

6. Between 12:00 noon and 12:00 midnight, and counting both starting and finishing positions, how many times will the hour and minute hands of a clock line up with each other?

Examples:

7. Ordinary dice are cubes, each with 6 square faces numbered 1 to 6.

Octahedral dice are regular geometric solids with 8 faces. Each face is an equilateral triangle, numbered 1 to 8.

When you roll an octahedral die, each number has an equal 1 in 8 chance of being uppermost when it comes to rest.

(a) When you roll two ordinary cubic dice together, the most frequent score is 7. What score will come up most frequently when you repeatedly roll two of these 8 octahedral dice and add the two top digits?

(b) What is the chance of throwing a score of 16 with one throw of two such dice?
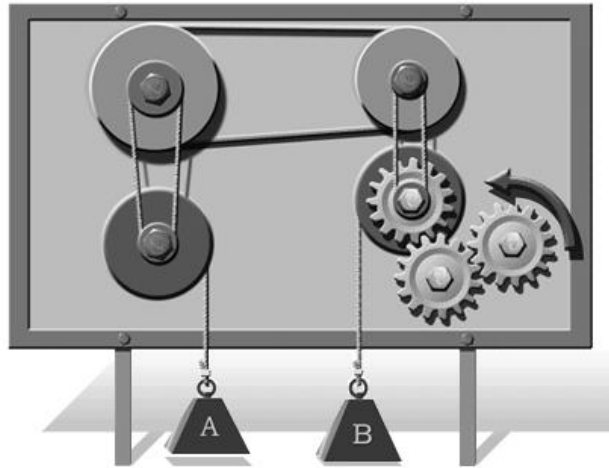
8. What is 1/2 of 2/3 of 3/4 of 4/5 of 5/6 of 6/7 of 7/8 of 8/9 of 9/10 of 1000?

The answer might surprise you!

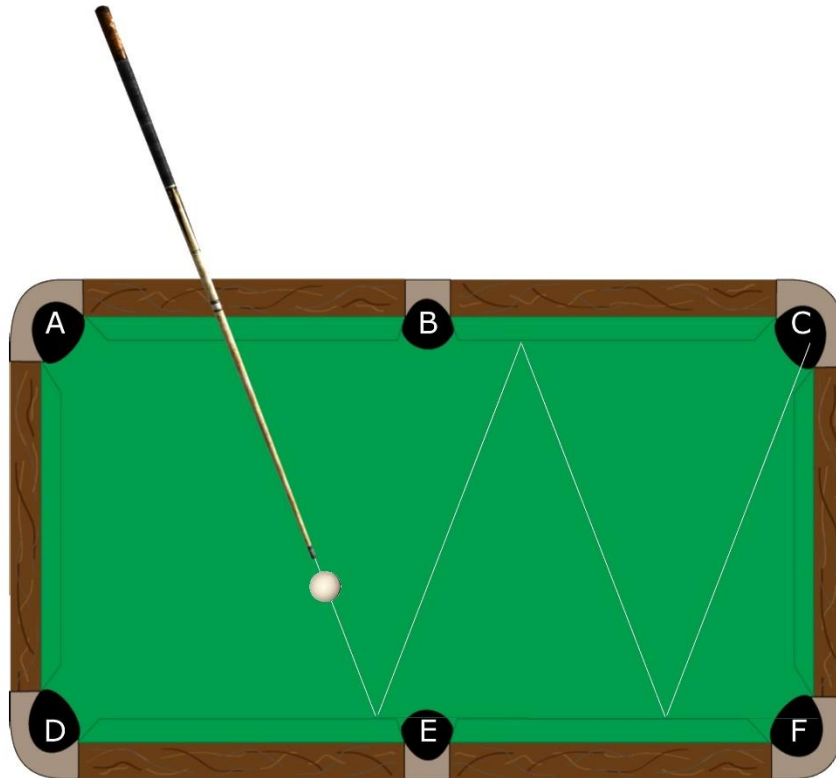Hint: Look for an easy way of working this out.

9.

Which direction will each weight travel (up or down) when the cogwheel on the right is turned anti-clockwise?

10. Write the number 23 in binary form

Solutions:

1. Pocket C

2. Ace of spades, Queen of Clubs, King of Spades

3. Yes:
You may find it useful to apply a series of "sieves" of ascending prime number factors
Is 157 divisible by 2? NO because it's an odd number
Is 157 divisible by 3? NO. If it was, its digits would add up to 3, 6 or 9
Is 157 divisible by 5? NO. It doesn't end in 5 (or 0)
Is 157 divisible by 7? NO. There's no easy trick but it's not a difficult calculation
Is 157 divisible by 11? NO.
There is no need to go further; why? Because the next possible prime number factor is
13. 13^2 = 169 which is higher than 157. If 157 had been divisible by 13 we would
already have found that out.

4. 1

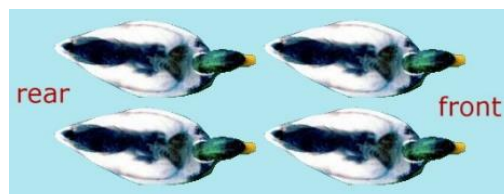three to the power of two minus two to the power of three

$= 3^2 - 2^3$

$= (3 \times 3) - (2 \times 2 \times 2)$

$= 9 - 8$

$= 1$

5. Four ducks
Providing they are arrayed as shown

6. 23 times

This can be easily demonstrated using the hand-setting controls on a clock. However, to explain why it might be so through logic: The examples should remind you that every time the minute hand lines up with the hour hand it has travelled slightly further than 30 minutes since the previous alignment, since the hour hand is also advancing.

So from going from 12:00 noon to 12:00 midnight, the hour hand requires one revolution; the minute hand makes twelve revolutions
BUT
The minute hand only "laps" the hour hand ELEVEN times, not twelve.

The question asks how often the hands form a straight line. When considering both situations for forming a straight line, (the minute hand overlapping the hour hand and the two hands pointing directly away from each other) this happens 23 times if you count both the noon and midnight positions. The two hands are aligned at noon and align another 22 times in the 12 hours to midnight.

7. (a) 9   (b) 1 in 64

This table shows all the possible numerical outcomes for rolling two octahedral dice and adding the individual scores

	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

There are 8 possible ways out of 64 of scoring a 9. That's the outcome with the highest frequency, or chance of occurring.

There is only one possible way out of 64 of scoring a 16.

8. 100
In arithmetic, "of" means "times" or "multiplied by"
When set out as numerator over denominator, there is a lot of CANCELLATION possible.

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{4}{5} \text{ of } \frac{5}{6} \text{ of } \frac{6}{7} \text{ of } \frac{7}{8} \text{ of } \frac{8}{9} \text{ of } \frac{9}{10} \text{ of } 1000$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9} \times \frac{9}{10} \times \frac{1000}{1}$$

$$= \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 1000}{2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}$$

$$= \frac{\cancel{2} \times \cancel{3} \times \cancel{4} \times \cancel{5} \times \cancel{6} \times \cancel{7} \times \cancel{8} \times \cancel{9} \times \cancel{1000}^{100}}{\cancel{2} \times \cancel{3} \times \cancel{4} \times \cancel{5} \times \cancel{6} \times \cancel{7} \times \cancel{8} \times \cancel{9} \times \cancel{10}_{1}}$$

$$= 100$$

9. "A" will rise and "B" will descend.

10. 10111

In binary notation 01 is equivalent to 1 in decimal

10	2 or 2^1
11	3
100	4 or 2^2
1000	8 or 2^3
10000	16 or 2^4
100000	32 or 2^5

And so on

Each place in a binary number as you move to the left can be described as the next higher power of 2, much as each place in a decimal number as you move to the left can be described as the next higher power of 10

The decimal number 23 can be expressed in powers of 2 as

$(1 \times 16) + (0 \times 8) + (1 \times 4) + (1 \times 2) + (1 \times 1)$

$= (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

And can hence be rendered in binary as 1 0 1 1 1