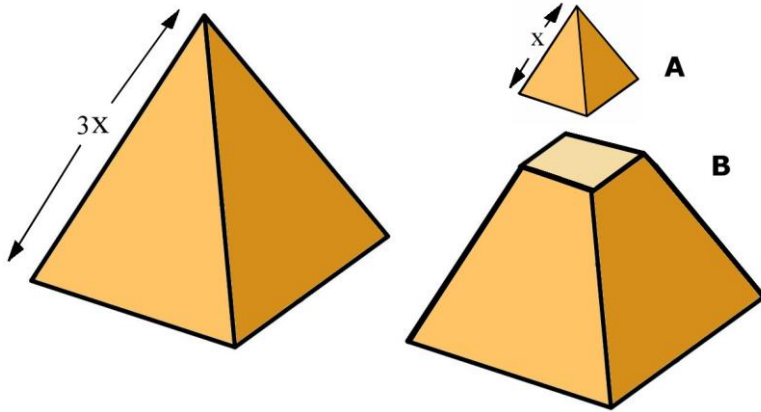




Logic and Maths Puzzles # 71 June 2018

1.



A rectangular pyramid resting on its base is cut horizontally into two pieces, a smaller pyramid **A** and a frustum **B**.

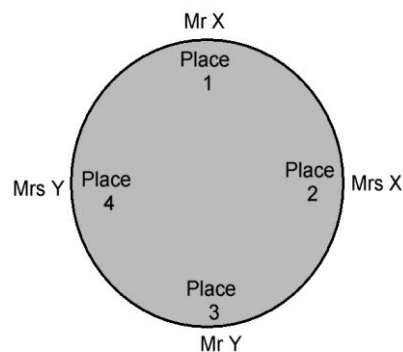
The cut is made at one third of the height of the pyramid down from its apex.

How many times greater is the volume of **B** than the volume of **A**?

2. How many farthings were in a British guinea? Given: that there were four farthings to the penny, twelve pence to the shilling, and twenty shillings to the pound. A guinea was equivalent to one pound and one shilling.

3. A party of four, consisting of two married couples, wants to share the same round table set for four people, so that each man sits next to his wife.

How many different ways is this possible? The diagram shows one possible arrangement.



4. At a recent visit to the reptile house at a local zoo, I counted a total of 27 heads and 70 feet. I was counting snakes, lizards and people. There were twice as many lizards as there were people.

- How many snakes were there?
- How many lizards were there?
- How many people were there?

5. Given: **$1 + 2 = 3$**

$1 + 2 + 3 = 6$

$1 + 2 + 3 + 4 = 10$

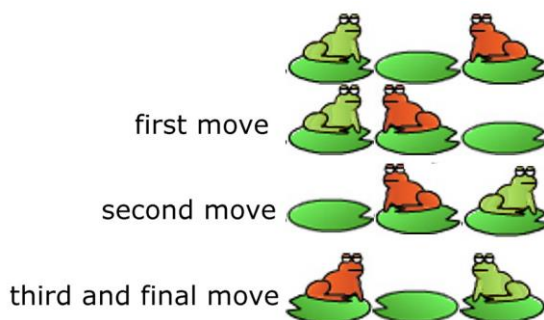
Can you see a pattern which will allow you to quickly calculate the sum of any number of consecutive whole numbers, starting with 1?

Use that pattern to calculate the sum of all whole numbers between 1 and 50 inclusive.

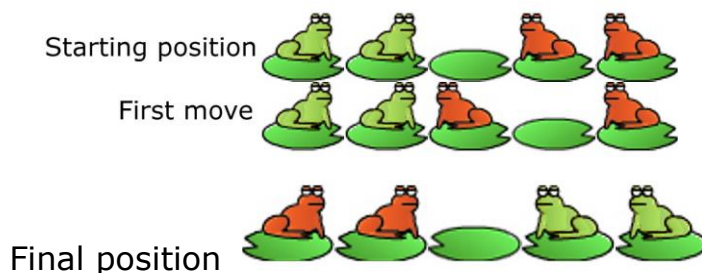
6. You have green frogs on one side of a pond, and an equal number of red frogs on the other. Each is sitting on its own lily pad, waiting to cross.

- There is 1 vacant lily pad between them.
- The frogs move by hopping onto the vacant lily pad, one at a time.
- A frog may only hop onto the vacant lily pad if it is adjacent OR if it only needs to jump over one other frog to reach the vacant lily pad.
- Frogs may not jump backwards

Example: 1 frog on each side



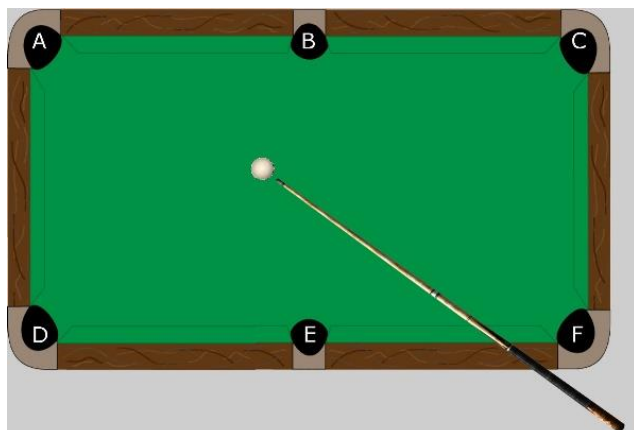
a. Following the same rules, what is the minimum number of moves required for 2 frogs on each side to exchange positions?



b. Following the same rules, what is the minimum number of moves required for 3 frogs on each side of the pond to change position, that is to go from



c. Now **predict** how many moves are needed to exchange 4 frogs on either side.

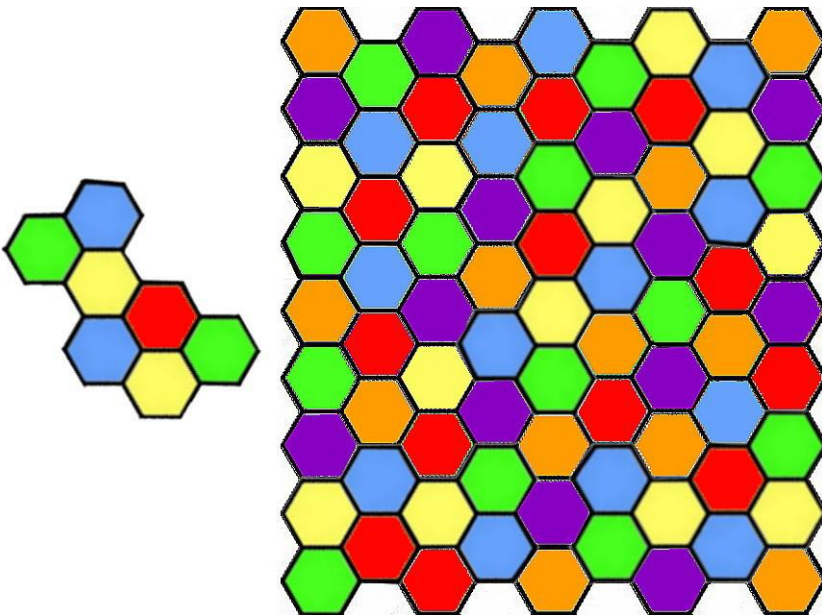


7. The diagram represents a billiard table with a cue lined up to strike the ball. Assuming the cue ball is struck cleanly, the ball is struck with the correct weight and the cushions play true, which pocket will the cue ball end up in?

8. The picture shows seven cards dealt face-up. Which cards were dealt first, third and fifth?

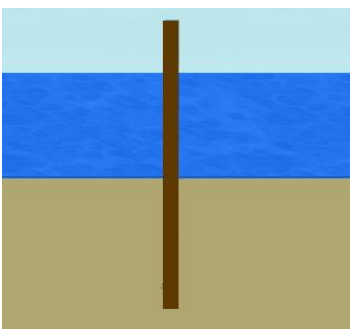


9.



How many times can the specific pattern of seven coloured hexagons on the left be found within the large array of hexagons on the right?

The seven-hexagon pattern may be rotated in the plane of the page but not reflected ("flipped").



10.

A pole is placed in a lake so that half the pole is driven into the solid bottom of the lake, one third is covered by water, and the remaining 12 metres sticks up into the air. How long is the pole?

Solutions:

1. B has 26 times the volume of A

The original pyramid and pyramid **A** are SIMILAR. That means that for every dimension in pyramid **A**, the same dimension in the original pyramid is exactly three times as long. The formula for the volume of a pyramid is

$$V = \frac{Lwh}{3}$$

Let the vertical height of the pyramid **A** = h units; then the vertical height of the original pyramid = 3h units

Let the width of the base of pyramid **A** = w units; then the width of the base of the original pyramid is 3w units

Let the length of the base of pyramid **A** = L units; then the length of the base of the original pyramid is 3L units

Volume of pyramid **A** = = Lwh/3 cubic units;

Volume of the original pyramid is (3L X 3w X 3h) / 3 cubic units = 27Lwh/3 cubic units

The volume of pyramid **A** is 1/27 that of the original pyramid. Frustrum **B** therefore accounts for the other 26/27 of the volume of the original pyramid

B has 26 times the volume of **A**

2. 1,008

One guinea is equivalent to 1 pound 1 shilling

= 21 shillings

= 21 X 12 = 252 pence

= 21 X 12 X 4 = 1,008 farthings

3. 16 ways: Any one of the four can occupy place 1, 2, 3 or 4

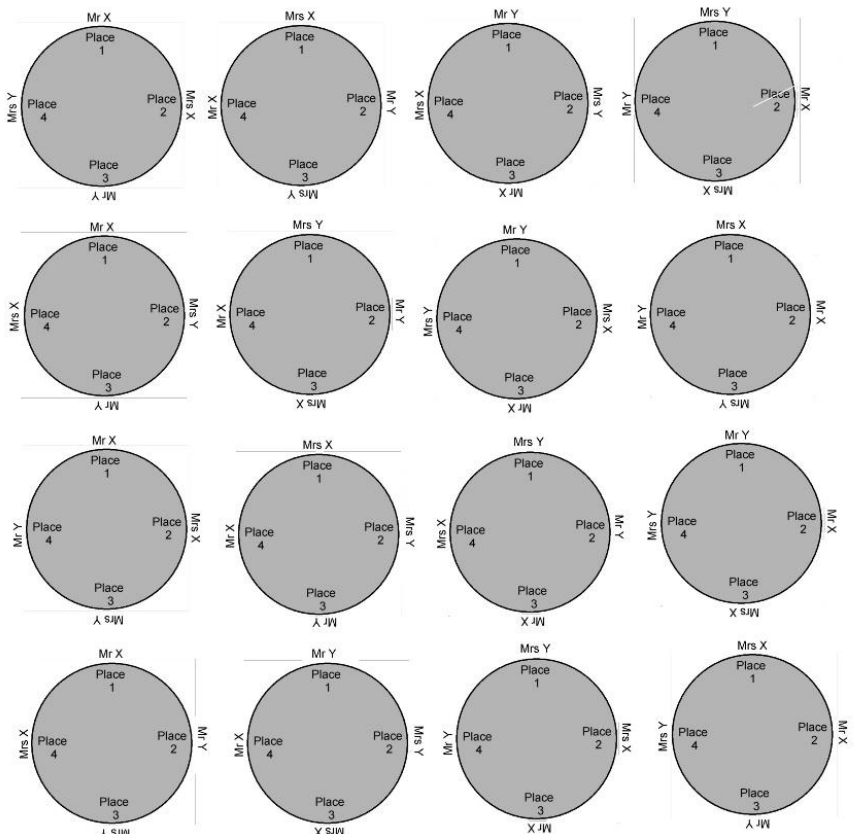
Let's say Mr X is in place 1; Mrs X may then occupy either place 2 or place 4.

Let's say she occupies place 2.

That leaves places 3 and 4 vacant

If Mr Y occupies place 3, then Mrs Y occupies place 4.

That's just one of 16 possible variations (see diagram)



4. a. 6 snakes b. 14 lizards c. 7 people

This could be solved by trial-and-error or with simultaneous equations.

Trial & Error			Simultaneous equations
	Heads (27)	Feet (70)	
1 person, 2 Lizards, 24 snakes	✓	10 NO	Let the number of people = p Let the number of lizards = l Let the number of snakes = s $p + l + s = 27$
2 people, 4 lizards, 21 snakes	✓	20 NO	$2p + 4l = 70$
3 people, 6 lizards, 18 snakes	✓	30 NO	But $l = 2p$
4 people, 8 lizards, 15 snakes	✓	40 NO	$2p + (4 \times 2p) = 70$
5 people, 10 lizards, 12 snakes	✓	50 NO	$2p + 8p = 70$
6 people, 12 lizards, 9 snakes	✓	60 NO	$10p = 70$
7 people, 14 lizards, 6 snakes	✓	70 YES	$p = 7$ therefore $l = 14$ $p + l + s = 27$ $s = 27 - 14 - 7 = 6$

5. The pattern can be written

$$\Sigma k = [n + 1]n / 2$$

where Σk is sum of the first n natural numbers. (i.e. the integers counting from 1 to n)

In words,

Multiply (the number you are counting up to) by (the number you are counting up to plus 1), then divide that by two.

The sum of all the whole numbers between 1 and 50 = **1275**

example: when counting from 1 to 3

$$\Sigma k = [3 + 1]3 / 2 = (4 \times 3) / 2 = 6$$

when counting from 1 to 4

$$\Sigma k = [4 + 1]4 / 2 = (5 \times 4) / 2 = 10$$

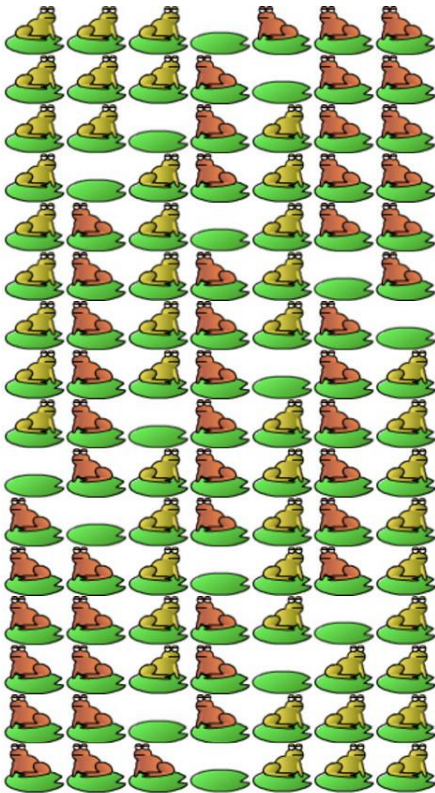
When counting from 1 to 5

$$\Sigma k = [5 + 1]5 / 2 = (6 \times 5) / 2 = 15$$

When counting from 1 to 50

$$\Sigma k = [50 + 1]50 / 2 = (51 \times 50) / 2 = \mathbf{1275}$$

6. a. 8 moves b. 15 moves c. 24 moves



The solution shown is for 3 frogs on either side. (If you understand the rules and the examples, it should not be hard to find the answer for 2 frogs without further help)

Once you have built up the sequence:

[0 frogs each side require 0 moves]

1 frog each side requires 3 moves

2 frogs each side requires 8 moves

3 frogs each side requires 15 moves

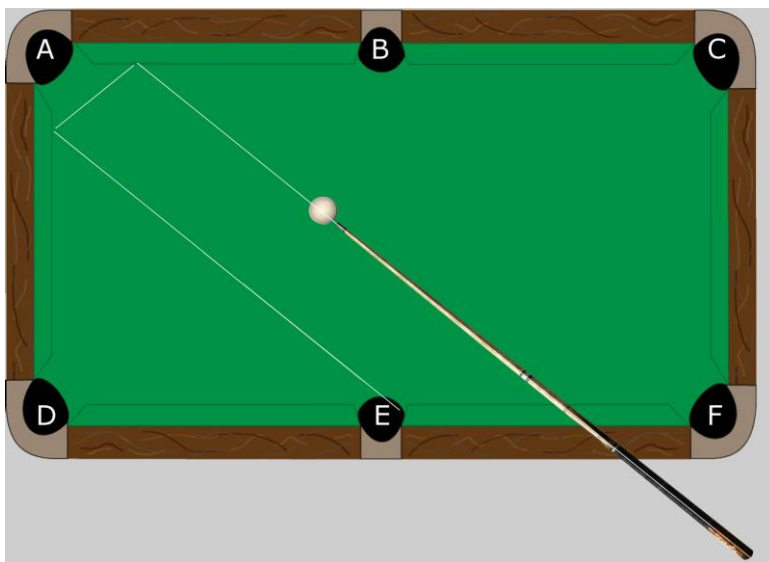
Note that each time you add a frog to each side the number of moves required for them to swap places increases by the next higher odd number

So swapping **4** frogs on each side will require $(15 + 9) =$ **24** moves

Swapping five frogs on each side will require $(24 + 11) =$ 33 moves

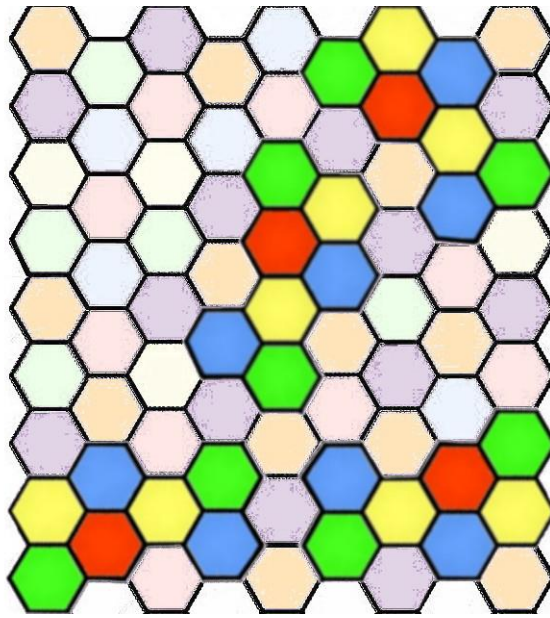
And so on.

7. Pocket E. It is based on the angle at which the ball strikes each cushion being the same as the reflected angle from that cushion. (see next page)



8. In order: Jack of Hearts, 7 of Diamonds, Queen of Hearts

9. Four times (see next page)



10. 72 metres

Let the length of the pole be l metres.

$$\frac{l}{2} + \frac{l}{3} + 12 = l$$

$$\frac{3l+2l}{6} + 12 = l$$

$$\frac{5l}{6} + 12 = l$$

$$l - \frac{5l}{6} = 12$$

$$\frac{l}{6} = 12$$

$$l = \mathbf{72}$$